

MAC - 2233 Average and Marginal Business and Economics Functions

Marginal Cost – The rate of change of the cost of producing x items: $C'(x)$

Average Cost – The average cost of each item among x items: $\bar{C}(x) = \frac{C(x)}{x}$

Marginal Average Cost – The rate of change of the average cost of each item among x items:

$$\bar{C}'(x)$$

Example: Bill owns a small business for selling pizza. The cost of producing x pizzas is given by the function $C(x) = .5x^2 - 4x + 8$

a) Find the approximate cost of producing the 11th pizza:

i) Find $C'(x) = 0.1x - 4$

ii) Find $C'(10) = 10 - 4 = \$6$ = The approximate cost of producing the 11th pizza.

b) Find the average cost per pizza for producing 20 pizzas:

i) Find $\bar{C}(x) = \frac{C(x)}{x} = \frac{0.5x^2}{x} - \frac{4x}{x} + \frac{8}{x} = 0.5x - 4 + \frac{8}{x}$

ii) Find $\bar{C}(20) = 0.5(20) - 4 + \frac{8}{20} = 10 - 4 + 0.4 = \6.40 = The average cost per pizza for producing 20 pizzas.

c) Find the average cost of producing the 12th pizza:

i) Find $\bar{C}'(x) = 0.5 - \frac{8}{x^2}$

ii) Find $\bar{C}'(11) = 0.5 - \frac{8}{11^2} = 0.5 - \frac{8}{121} = 0.5 - .066 = \0.43 = The average cost per pizza for producing the 12 pizza.

Marginal Revenue – The rate of change for the revenue for producing x items: $R'(x)$

Average Revenue – The average revenue of each item from among x items: $\bar{R}(x) = \frac{R(x)}{x}$

Marginal Average Revenue – The rate of change of the average revenue of each item among x items:

$$\bar{R}'(x)$$

Example: Bill owns a small business for selling pizza. The revenue for producing x pizzas is given by the function $R(x) = -0.2x^2 + 2x$

a) Find the approximate revenue for producing the 8th pizza:

i) Find $R'(x) = -0.4x + 2$

ii) Find $R'(7) = -0.4(7) + 2 = -\$0.80$ = The approximate revenue for producing the 11th pizza.

b) Find the average revenue per pizza for producing 8 pizzas:

i) Find $\bar{R}(x) = \frac{R(x)}{x} = \frac{-0.2x^2}{x} + \frac{2x}{x} = -0.2x + 2$

ii) Find $\bar{R}(8) = -0.2(8) + 2 = -1.60 + 2 = \0.40 = The average revenue per pizza for producing 8 pizzas.

c) Find the average revenue for producing the 8th pizza:

i) Find $\bar{R}'(x) = -0.2$

ii) Find $\bar{R}'(7) = -\$0.20$ = The average revenue per pizza for producing the 8th pizza.

Marginal Profit – The rate of change for the profit for producing x items: $P'(x)$

Average Profit – The average profit of each item from among x items: $\bar{P}(x) = \frac{P(x)}{x}$

Marginal Average Profit – The rate of change of the average profit of each item among x items:

$$\bar{P}'(x)$$

Example: Bill owns a small business for selling pizza. The revenue for producing x pizzas is given by the function $P(x) = -0.7x^2 + 6x - 8$

a) Find the approximate profit for producing the 4th pizza:

i) Find $P'(x) = -1.4x + 6$

ii) Find $P'(3) = -1.4(3) + 6 = \$1.80$ = The approximate profit for producing the 4th pizza.

b) Find the average profit per pizza for producing 4 pizzas:

i) Find $\bar{P}(x) = \frac{P(x)}{x} = \frac{-0.7x^2}{x} + \frac{6x}{x} - \frac{8}{x} = -0.7x + 6 - \frac{8}{x}$

ii) Find $\bar{P}(4) = -0.7(4) + 6 - \frac{8}{4} = -2.8 + 6 - 2 = \1.20 = The average profit per pizza for producing 4 pizzas.

c) Find the average profit per pizza for producing the 6th pizza:

i) Find $\bar{P}'(x) = -0.7 - \frac{8}{x^2}$

ii) Find $\bar{P}'(5) = -0.7 - \frac{8}{5^2} = -0.7 - \frac{8}{25} = -0.7 - 0.32 = -\1.02 = The average profit per pizza for producing the 6th pizza.

Practice Problems:

John owns a small business selling High Definition Televisions. His Cost, Revenue and Profit functions are given by the following:

$$R(x) = -0.05x^2 + 250x \quad C(x) = 125,000 + 35x \quad P(x) = -0.05x^2 + 215x - 125,000$$

- a) Find the cost of producing the 1001th TV.
- b) Find the average cost per TV for producing 1000 TVs.
- c) Find the average cost per TV for producing the 1001th TV.
- d) Find the revenue for producing the 1001th TV.
- e) Find the average revenue per TV for producing 1000 TVs.
- f) Find the average revenue per TV for producing the 1001th TV.
- g) Find the profit for producing the 1001th TV.
- h) Find the average profit per TV for producing 1000 TVs.
- i) Find the average profit per TV for producing the 1001th TV.

Answer Key:

- a) \$35
- b) \$160
- c) \$.035
- d) \$150.10
- e) \$200
- f) \$0.15
- g) \$115.10
- h) \$40
- i) \$.115