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LEARNING COMMONS

Solving Quadratic Equations

In this lab assignment we will be solving equations that are called quadratic equations. A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$. A quadratic equation is in **standard form** when the polynomial is in descending order and equal to zero.

To solve an equation by factoring we will be using the following principle.

Principle of Zero Products

If $a \cdot b = 0$
then $a = 0$ or $b = 0$

This principle states that if you have a product equal to zero, then at least one of the factors must equal zero (since $0 \cdot \text{anything} = 0$).

Steps for Solving Equations by Factoring

1. Put the equation in standard form. (The polynomial must be in descending order and equal to zero.)
2. Factor the polynomial
3. Set each factor equal to 0
4. Solve each equation

Example: Solve: $(y + 8)(y - 5) = 0$
Question to ask: Is the equation in standard form?
No, but that is because this example has already been factored for us. It is not in descending order, but it is equal to 0.

***You can tell a polynomial has been factored if it is written as a multiplication problem—a product of binomials.**
$(y + 8)(y - 5)$ is the polynomial in factored form. So at this point we can set each of the two factors equal to 0.

$$\begin{array}{rcl} y+8=0 & \qquad & y-5=0 \\ -8\ \ -8 & & +5\ \ +5 \\ \hline y=-8 & & y=5 \end{array}$$

The solutions are -8, and 5. To check, plug in the solutions for y and see if we get a true statement.

$$\begin{array}{ll} (-8+8)(-8-5) & \qquad (5+8)(5-5) \\ =0\cdot(-13) & \qquad =13\cdot 0 \\ =0 & \qquad =0 \end{array}$$

Example: Solve: $x^2 + 6x + 8 = 0$
Question to ask: Is the equation in standard form? Yes.
Factor the polynomial by finding the factors of 8 that add up to 6.

$$x^2 + 6x + 8 = 0$$
$$(x + 4)(x + 2) = 0$$

Set each factor equal to 0 and solve each equation.

$$\begin{array}{r} x+4=0 \\ -4\ \ -4 \\ \hline x=-4 \end{array} \qquad \begin{array}{r} x+2=0 \\ -2\ \ -2 \\ \hline x=-2 \end{array}$$

The solutions are –4 and –2.

Example: Solve: $a^2 - 5a = 24$
Question to ask: Is the equation in standard form?
No—the equation is not equal to 0. The first step is to subtract 24 from each side of the equation so that the equation is equal to 0.

$$\begin{array}{r} a^2-5a=24 \\ -24\ \ -24 \\ \hline a^2-5a-24=0 \end{array}$$

NOTE: The –24 is placed behind the $-5a$ so that the polynomial is in descending order. Now factor the polynomial.

$$a^2 - 5a - 24 = 0$$

Find the factors of –24 that add up to –5.

$$(a - 8)(a + 3) = 0$$

Set each factor equal to 0 and solve each equation.

$$\begin{array}{r} a-8=0 \\ +8\ \ +8 \\ \hline a=8 \end{array} \qquad \begin{array}{r} a+3=0 \\ -3\ \ -3 \\ \hline a=-3 \end{array}$$

Example: Solve: $b^2 + 20 = 9b$

First you must put the equation in standard form by subtracting $9b$ from each side of the equation.

$$\begin{array}{r} b^2 + 20 = 9b \\ -9b \quad -9b \\ \hline b^2 - 9b + 20 = 0 \end{array}$$

Be sure to put the polynomial in descending order. Be careful to keep the signs with the terms.

$$b^2 - 9b + 20 = 0$$

Now factor the polynomial by finding the factors of 20 that add up to –9.

$$(b - 4)(b - 5) = 0$$

Set each factor equal to 0 and solve each equation.

$$\begin{array}{r} b - 4 = 0 \\ +4 \ +4 \\ \hline b = 4 \end{array} \qquad \begin{array}{r} b - 5 = 0 \\ +5 \ +5 \\ \hline b = 5 \end{array}$$

The solutions are 4 and 5.

Example: Solve: $x(x - 11) = 12$

In this problem we first have to subtract 12 from each side of the equation. Note that the 12 is NOT combined with the −11 on the left side of the equation. You cannot combine the two numbers because the −11 is attached to the x in the parentheses.

$$\begin{array}{r} x(x - 11) = 12 \\ -12 \quad -12 \\ \hline x(x - 11) - 12 = 0 \end{array}$$

The equation is now equal to 0, but we need to distribute the x to get the polynomial into descending order.

$$x^2 - 11x - 12 = 0$$

Now that the equation is in standard form, factor the polynomial.

$$(x - 12)(x + 1) = 0$$

Set each factor equal to 0 and solve each equation.

$$\begin{array}{rl} x-12=0 & \qquad x+1=0 \\ +12\ +12 & \qquad -1\ -1 \\ \hline x=12 & \qquad x=-1 \end{array}$$

The solutions are 12 and −1.

Example: Solve: $(y + 3)(y + 10) = -10$

This example is similar to the first example we did, but with a big difference. The left hand side of the equation is in factored form, but the equation is NOT equal to zero. So we cannot start setting those two factors equal to zero at this point. First, we need to put the equation in standard form by adding 10 to each side of the equation.

$$\begin{array}{r} (y+3)(y+10)=-10 \\ +10 \quad +10 \\ \hline (y+3)(y+10)+10=0 \end{array}$$

The 10 does not combine with the 10 or 3 in the parentheses. Now you will need to use the FOIL method to multiply the two binomials.

$$y^2 + 10y + 3y + 30 + 10 = 0$$

Combine the like terms. $y^2 + 13y + 40 = 0$

Now the equation is in standard form. The next step is to factor the polynomial.

$$(y + 8)(y + 5) = 0$$

Set each factor equal to 0 and solve each equation.

$$\begin{array}{rl} y+8=0 & \qquad y+5=0 \\ -8\ -8 & \qquad -5\ -5 \\ \hline y=-8 & \qquad y=-5 \end{array}$$

The solutions are −8 and −5.

Now you try these problems.

Exercises: Solve by factoring.

a. $(x + 7)(x - 4) = 0$

f. $2x^2 + x = 6$

b. $4y(y + 3) = 0$

g. $x^2 + 2x = 35$

c. $a^2 + 9a + 14 = 0$

h. $x(x + 3) = 28$

d. $x^2 - 16 = 0$

i. $y^2 - 7y = 8$

e. $3a^2 + 14a + 8 = 0$

j. $(x + 4)(x - 1) = 14$

KEY

a. $-7, 4$	c. $-7, -2$	e. $\frac{-2}{3}, -4$	g. $-7, 5$	i. $8, -1$
b. $0, -3$	d. $-4, 4$	f. $\frac{3}{2}, -2$	h. $4, -7$	j. $-6, 3$

This instructional aid was prepared by the Tallahassee Community College Learning Commons.