# How to find an Inverse Function

**Inverse Function –** A function derived from an original function in which each input becomes an output and each output because an input for the function.

## Example 1:    Inverse Relations:

**Find the inverse of the relation:**
$\{(1,2), (2,4), (7,-2), (9,-3), (10,6)\}$

## Solution:

Interchange each x and y value and rewrite the relation:
$\{(\mathbf{2,1}), (\mathbf{4,2}), (\mathbf{-2,7}), (\mathbf{-3,9}), (\mathbf{6,10})\}$

## Example 2:  Inverse Functions

**Find the inverse of the following function:**
$f(x) = 8x + 9$

## Solution:

**Step 1:**  Replace f(x) with y:
y = 8x + 9
**Step 2:**  Interchange x and y:
x = 8y + 9
**Step 3:**  Solve for y (Subtract 9 from each side):
$x - \mathbf{9} = 8y + 9 - \mathbf{9}$    →    $x - 9 = 8y$

**Step 4:**  Divide by 8 on each side:
$\frac{x-9}{8} = \frac{8y}{8}$    →    $\frac{x-9}{8} = y$

**Step 4:**  Replace y with $f^{-1}(x)$:
$$\mathbf{f^{-1}(x)} = \frac{\mathbf{x-9}}{\mathbf{8}}$$

## Example 3:

**Find the inverse of the following function:**

$f(x) = \dfrac{2}{x+3}$

**Step 1:**  Replace f(x) with y :

$y = \dfrac{2}{x+3}$

**Step 2:**  Interchange x and y:

$x = \dfrac{2}{y+3}$

**Step 3:** Solve for y (Multiply by y + 3 on each side):

$x(y+3) = \frac{2(y+3)}{(y+3)}$        →        $x(y+3) = 2$

**Step 4:**  Divide by x on each side:

$\frac{x}{x}(y+3) = \frac{2}{x}$        →        $y+3 = \frac{2}{x}$

**Step 5:**  Subtract 3 on each side:

$y+3 - \mathbf{3} = \frac{2}{x} - \mathbf{3}$        →        $y = \frac{2}{x} - 3$

**Step 6:**  Replace y with $f^{-1}(x)$:

$\mathbf{f^{-1}(x)} = \frac{\mathbf{2}}{\mathbf{x}} - \mathbf{3}$

## Practice Problems

**Find the inverse of the following functions:**
1) $h(x) = x^2 - 4$
2) $f(x) = \frac{4x-1}{-2x+3}$
3) $g(x) = \sqrt{5x-6}$

**Solutions:**
1) 1) $h^{-1}(x) = \sqrt{x+4}$
2) $f^{-1}(x) = \frac{3x+1}{2x+4}$
3) $g^{-1}(x) = \frac{y^2+6}{5}$