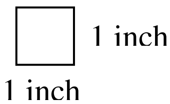
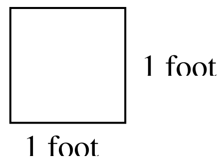
Area of Basic Geometric Figures

The area of a figure measures the surface of the figure. The unit of measure for area cannot be a linear unit. To measure area we use square units such as:

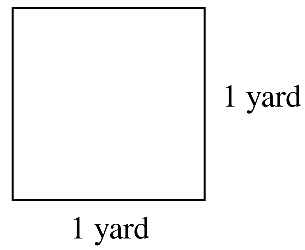
The Square Inch



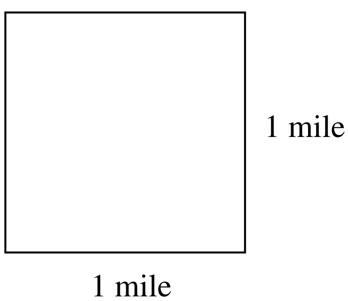
The Square Foot



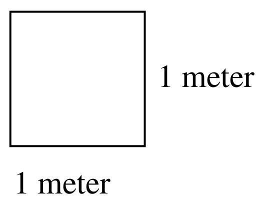
The Square Yard



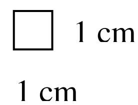
The Square Mile



The Square Meter



The Square Centimeter



The abbreviations that are used in mathematics are:

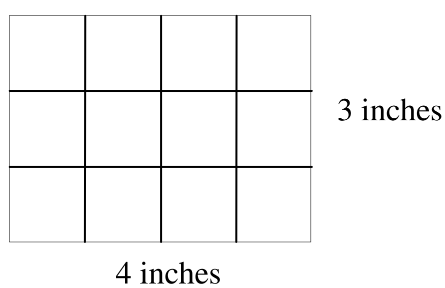
square inch: in² square yard: yd² square foot: ft²

square meter: m² square mile: mi² square centimeter: cm²

AREA FORMULAS: Memorize these four formulas.

A. RECTANGLE

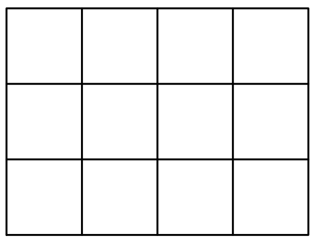
It is easy to understand the formula for the area of a rectangle. Find the area of the rectangle shown here.



This simply means: "how many square inches cover the surface of this rectangle?" We can draw lines at each inch along a width a length. We can then count the little squares that are formed. (What is the size of each little square?)

The area is the number of these square inches.

There are 12 squares. Each square is a square inch. The area is 12 square inches. We will write 12 in².(NOTICE this does <u>NOT</u> mean 12 × 12!) It is read 12 square inches; there are 12 of these little squares.



3 inches

4 inches

Area = 12 in²

> How could we have found 12 in² without having to draw the squares?
>
> 4 squares on each row × 3 rows = 12 squares.
>
> To find the area of the rectangle, we multiply the length by the width.

<u>Area of a Rectangle</u>:

AREA = LENGTH × WIDTH or A = LW

EXAMPLE: Find the area of the rectangle shown.

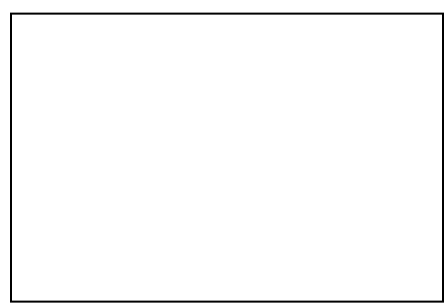
Area = length × width

$$\text{Area} = 8\frac{3}{4}\text{ ft} \times 6\frac{1}{2}\text{ ft}$$

$$= \frac{35}{4}\text{ ft} \times \frac{13}{2}\text{ ft}$$

$$= \frac{455}{8}\text{ ft}^2$$

$$= 56\frac{7}{8}\text{ ft}^2$$



$6\frac{1}{2}$ ft.

$8\frac{3}{4}$ ft.

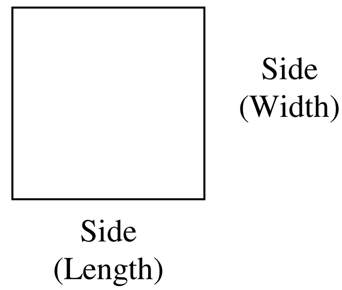
REMEMBER, area doesn't tell the length of a line segment; it tells how many <u>square</u> units cover this flat surface. The <u>area</u> is measured in <u>square units</u>.

B. SQUARE

REMEMBER a square is a special rectangle. We can use AREA = LENGTH × WIDTH. Since the length and width are the same, we call each one a side.

Area of a Square:

AREA = SIDE × SIDE
or AREA = $(\text{SIDE})^2$
or $A = s^2$



The area of a square which is 0.7 cm on each side is

$$\text{AREA} = \text{SIDE} \times \text{SIDE} = 0.7\text{cm} \times 0.7\text{cm} = 0.49\ \text{cm}^2$$

or

$$\text{AREA} = (\text{side})^2 = (0.7\ \text{cm})^2 = 0.49\ \text{cm}^2$$

Area = $0.49\ \text{cm}^2$

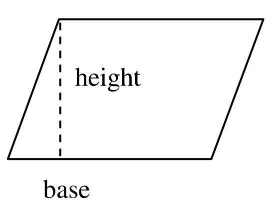
C. TRIANGLE

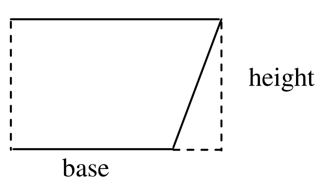
The area of a triangle is found by multiplying

$$\frac{1}{2} \times \text{base} \times \text{height}$$

EXPLANATION: If we cut this parallelogram along the dotted line, and place the small triangle at the other end, we will form a rectangle.

The length × width of the rectangle is the base × height.





The area of the parallelogram is base × height.

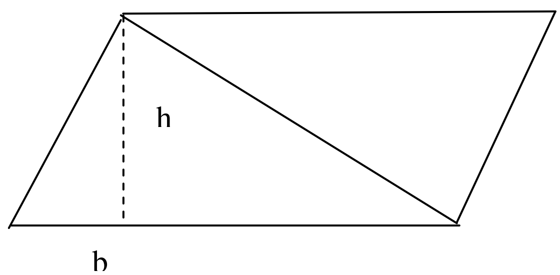
$$A = b \cdot h$$

Now if we draw and cut out two triangles that are exactly the same size, we can flip one over and place them to form a parallelogram.

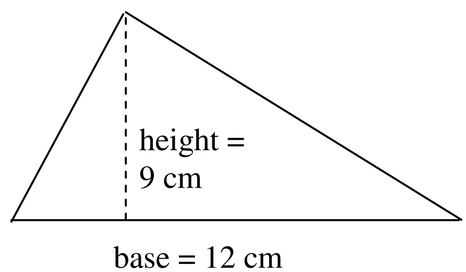
<u>The area of one triangle is half the area of the parallelogram.</u>

$$\text{AREA} = \frac{1}{2} \times \text{base} \times \text{height}$$

or $$A = \frac{1}{2} \times b \times h$$



EXAMPLE: Find the area of the triangle shown.

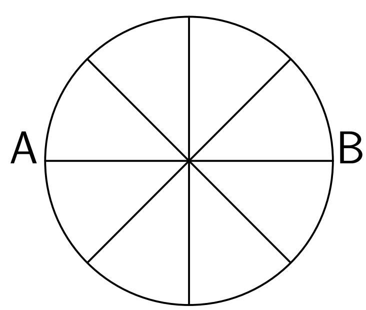


$$\text{AREA} = \frac{1}{2} \times 12\text{ cm} \times 9\text{ cm}$$

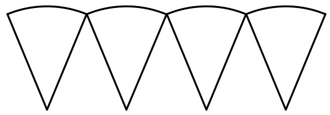
$$= 54\text{ cm}^2$$

Area = 54 cm^2

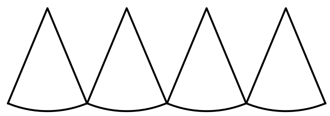
D. CIRCLE



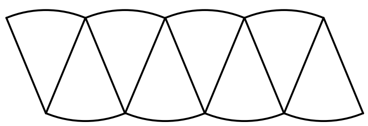
Imagine a pie cut into **<u>many</u>** little wedges. If we take the wedges above AB and place them like this



If we take the wedges below AB and place them like this

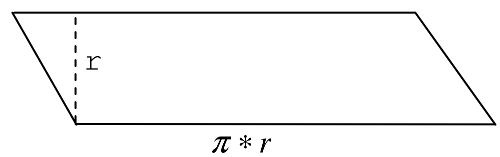


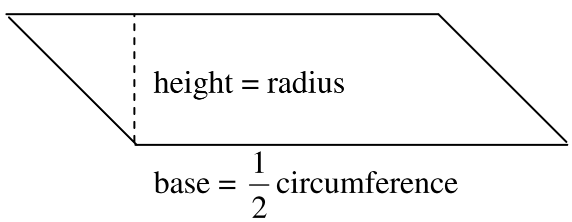
They can be placed together like this



The smaller the wedges, the straighter the edges would become.

Imagine now a parallelogram has been formed from these very small wedges placed together. (There are so many wedges that the sides <u>seem</u> to be straight.)





The base is ½ × the circumference of the original circle. The height is the radius of the circle.

The formula area = base × height

becomes area = (½ × circumference) × radius

REMEMBER the circumference = 2 × π × radius; this means

$$\text{Area} = \frac{1}{2} \times (2 \times \pi \times \text{radius}) \times \text{radius} \qquad \left(\frac{1}{2} \times 2 = 1\right) \text{ and } (1 \times \pi = \pi)$$

Regrouping, we get the formula for

Area of circle. Area = π × radius × radius or Area = π × (radius)² or $A = \pi r^2$

This instructional aid was prepared by the Tallahassee Community College Learning Commons. -

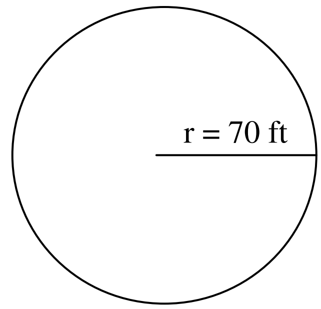
EXAMPLE: Find the area of a circle with a radius of 70 ft. Use $\pi = \frac{22}{7}$

$$A = \pi r^2$$

$$= \frac{22}{7} \times (70 \text{ ft})^2$$

$$= \frac{22}{\cancel{7}_{1}} \times \frac{\cancel{70}^{10}}{1} \text{ ft} \times \frac{70}{1} \text{ ft}$$

Area = $15{,}400 \text{ ft}^2$



Composite Geometic Figures

To find the area of composite figures, draw lines to form the figures whose areas you know. Find the area of each figure and add or subtract to get the area of the composite figure. It helps to plan your strategy **<u>in writing</u>** before you begin.

EXAMPLE 1: Find the area of the rectangle with a triangular region removed.

Area of composite figure =

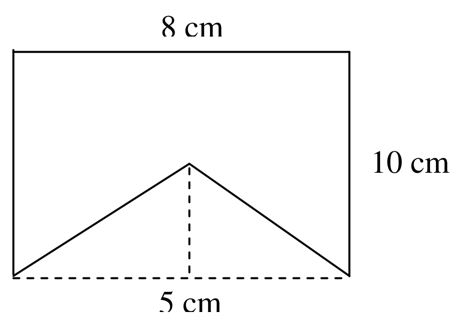
Area of rectangle – area of triangle

Area = (length × width) – (½ × base × height)

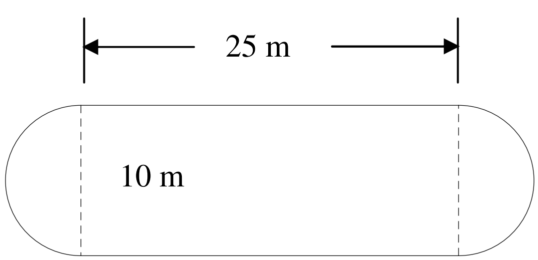
$= 10 \text{ cm} \times 8 \text{ cm} - \frac{1}{2} \times 5 \text{ cm} \times 8 \text{ cm}$

$= 80 \text{ cm}^2 - 20 \text{ cm}^2$

Area of composite figure = $80 \text{ cm}^2 - 20 \text{ cm}^2 = 60 \text{ cm}^2$

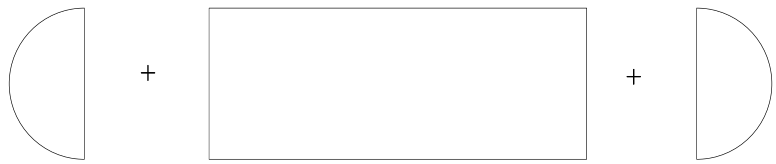


EXAMPLE 2: Find the area of the skating rink. Use $\pi = 3.14$

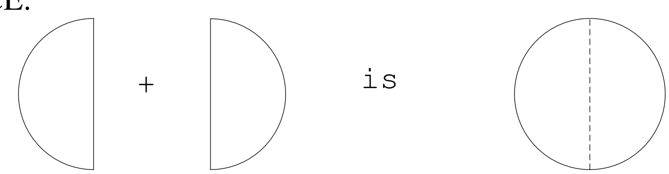


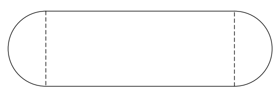
(Drawing is not to scale)

STRATEGY:



NOTICE:



The area of  is

Area of circle + Area of rectangle
$\pi \times (\text{radius})^2$ + length $\times$ width

Circle:

We know the diameter = 10 m
so radius = $\frac{1}{2} \times 10 = 5$ m
Area $= 3.14 \times (5)^2$
$= 3.14 \times 25$
$= 78.50$ m²

Rectangle:

$A = 10 \times 25$
$= 250$ m²

Area of the composite figure is 78.50 m² (circle) + 250.00 m² (rectangle)
= 328.50 m² composite figure

<u>APPLICATIONS:</u>

To determine the amount of carpeting you'll need for a room that is 12 ft long and 9 ft wide, you can find the area.

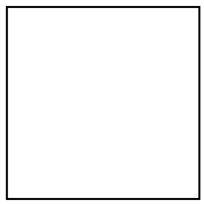
Area = length $\times$ width
= 12 ft $\times$ 9 ft
= 108 ft²

Carpeting is sold by the square yard. You can measure the room in yards.

Length = 12 ft = 4 yd
Width = 9 ft = 3 yd

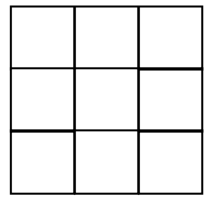
$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 4\text{ yd} \times 3\text{ yd} \\ &= 12\text{ yd}^2 \end{aligned}$$

Another way is to convert square feet to square yards using 1 yd^2 = 9 ft^2



1 yd

1 yd



1 yd = 3ft

1 yd = 3 ft

1 square yard = 9 square feet

Now multiply by “1”, where $1 = \dfrac{1\,\text{yd}^2}{9\,\text{ft}^2}$

$$\frac{108\text{ ft}^2}{1} \times \frac{1\,\text{yd}^2}{9\,\text{ft}^2} = 12\text{ yd}^2$$

(We find how many groups of 9 ft^2 are in 108 ft^2. That is how many square yards there will be in 108 ft^2).

PROBLEMS

DIRECTIONS

1. *Draw the figure and write in the given measures.*

2. *Write the area formula.*

3. *Replace the parts of the formula with the given measures and simplify.*

4. *The answer will be in square units!*

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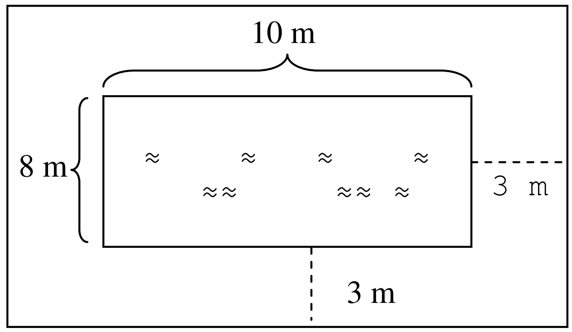
1. Find the area of a rectangle that is 4.6 m long and 1.4 m wide.

2. Find the area of a square that is ½ meter on each side.

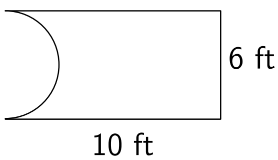
3. Find the area of a pizza that has a 12 inch diameter. Use $\pi = 3.14$.

4. Find the area of a triangle which has a base of 38 inches and a height of 14 inches.

5. A swimming pool is 10 m by 8 m. It is surrounded by a walkway that is 3 m wide. Find the area of the walkway. (HINT: Find the area of the large rectangle and subtract the area of the pool. What is the length of the large rectangle? What is its width?)



6. Find the area of this composite figure. Use $\pi = 3.14$



<u>ANSWER KEY:</u>

1. 6.44 m²
2. ¼ m²
3. 113.04 in²
4. 266 in²
5. 224 – 80 = 144 m²
6. 60 – 14.13 = 45.87 ft²