The T Confidence Interval of a Population Mean

T Intervals are similar to Z Intervals; however, instead of a Critical Z Value, a T Interval uses a Critical T Value, and uses the _sample_ standard deviation, s, instead of the population standard deviation, σ.

$$T\ Confidence\ Interval = \ \bar{x} \pm t \times \frac{s}{\sqrt{n}}$$

Consider this example: The Interdepartmental Delivery Service coordinator wants to estimate the true mean number of interdepartmental letters employees receive per week. She takes a sample of 26 employees and obtains a mean of 6 letters per week with a standard deviation of 2.3 letters. She needs to be 99% confident of her estimate. Construct the Confidence Interval for the true mean for interdepartmental letters delivery per week.

Step 1: Find the Standard Error:

$$S.E. = \frac{s}{\sqrt{n}} \qquad\qquad S.E. = \frac{2.3}{\sqrt{26}} \approx 0.451$$

Step 2: Select Critical Value:

To find the Critical Value of a T distribution it will be easiest to use the Table at the end of a textbook (your teacher will provide a T table for tests, and exam). The Degrees of Freedom is calculated by subtracting one from the Sample Size; $DF = \ n - 1 = 26 - 1 = 25$. Match the Confidence Column with the Row that has the Degrees of Freedom.

Central Area		.80	.90	.95	.98	.99
Confidence Level		80%	90%	95%	98%	99%
Degrees of Freedom.	21	1.32	1.72	2.08	2.52	2.83
n-1	22	1.32	1.72	2.07	2.51	2.82
	23	1.32	1.71	2.07	2.5	2.81
	24	1.32	1.71	2.06	2.49	2.8
	25	1.32	1.71	2.06	2.49	2.79

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Step 3: Compute the Margin of Error:

$$Margin\ of\ Error = t \times S.E.$$

$$Margin\ of\ Error = 2.79 \times 0.451$$

$$Margin\ of\ Error = 1.258$$

Step 4: Write out the Confidence Interval:

$$C.I. = \bar{x} \pm M.o.E.$$

$$C.I. = 6 \pm 1.258$$

$$(6 - 1.258, 6 + 1.258)$$

$$(4.742, 7.258)$$

Step 5: Interpret the Confidence Interval:

"The Coordinator can be 99% confident that the average number of interdepartmental employee letters is between 4.742 and 7.258 letters per week."

Use the Texas Instruments calculator to find the confidence interval

Calculator Steps:	The Output:
STAT > TESTS > 8: TInterval…	(4.7427, 7.2573)
Inpt: Stats	$\bar{x}$: 6
$\bar{x}$: 6	Sx = 2.3
Sx : 2.3	n = 26
n : 26	
C-Level: .99	
Calculate:	