WILLIAM D. LAW, JR. LEARNING COMMONS

Polar and Rectangular Coordinate Conversions

Polar Coordinate System – Any ordered pair written in the form of (r, θ) where r is the r radius from the Origin point O to a fixed point P and θ is the angle between the Polar Axis and the segment $\overline{OP}$.

Rectangular Coordinate System – Any ordered pair that can be written in the form of (x, y) where x is the horizontal component and y is the vertical component of the point.

$x = r\cos\theta$ and $y = r\sin\theta$

Converting from Polar to Rectangular Coordinates:

Example: Find the Rectangular Coordinates for the point that has Polar Coordinates $(2, 60°)$.

Solution: $x = r\cos\theta$ and $y = r\sin\theta$

$x = 2\cos 60°$	$y = 2\sin 60°$
$= 2 \times \frac{1}{2}$	$= 2 \times \frac{\sqrt{3}}{2}$
$\mathbf{= 1}$	$\mathbf{= \sqrt{3}}$

The Rectangular Coordinates for the point that has Polar Coordinates $(2, 60°)$ is $(\mathbf{1}, \mathbf{\sqrt{3}})$

Converting from Polar Coordinates to Rectangular Coordinates:

Given $r^2 = x^2 + y^2$ and $\tan\theta = \frac{y}{x}$

Example: Find the Polar Coordinates for the point that has Rectangular Coordinates $(3, 3)$.

Solution: $r^2 = x^2 + y^2$

Given: $r^2 = 3^2 + 3^2$	$\tan\theta = \frac{y}{x}$
$r^2 = 9 + 9$	$\tan\theta = \frac{3}{3}$
$r^2 = 18$	$\tan\theta = 1$
$r = \sqrt{18} = 3\sqrt{2}$	$\tan^{-1}(1) = 45°$

The Polar Coordinates for the point that has Rectangular Coordinates $(3, 3)$ is $(\mathbf{3\sqrt{2}}, \mathbf{45°})$.

Example: Express the following equations in Polar coordinates (Solve for r): $y^2 = 2x$

Solution:

Step 1: $\mathbf{y^2 = (r\sin\theta)^2}$ **and** $\mathbf{2x = 2r\cos\theta}$

Step 2: $r^2(\sin\theta)^2 = 2r\cos\theta$

Step 3: Solve for r: $r = \frac{2\cos\theta}{(\sin\theta)^2}$

$r = 2\frac{\cos\theta}{\sin\theta}\frac{1}{\sin\theta}$ $\quad$ $\mathbf{r = 2\cot\theta\csc\theta}$

Example: Express the following Polar equations in Rectangular Coordinates: $r = 5\csc\theta$

Solution:

Step 1: $r = \frac{5}{\sin\theta}$

Step 2: $r\sin\theta = 5$

Step 3: $y = r\sin\theta = 5$ $\quad$ **y= 5**

Practice Exercises:

Find the rectangular coordinates for the point that has the given polar coordinates (Round to two decimal places):

1) $(4, 80°)$ 2) $(-2, 150°)$ 3) $(7, 33°)$

Find the polar coordinates for the point that has the given rectangular coordinates (Round to two decimal places):

4) $(-3, 4)$ 5) $(10, -2)$ 6) $(5, 7)$

Express the following equation in Polar coordinates:

7) $2x^2 = y$

Express the Polar Equation in Rectangular Coordinates:

8) $r = 4\csc\theta$

Solutions:

1) $(0.69, 3.94)$ 2) $(1.73, -1)$ 3) $(5.87, 3.81)$ 4) $(5, 126.87)$

5) $(10.20, 149.97)$ 6) $(8.60, 54.46)$ 7) $r = \frac{1}{2}\tan\theta\sec\theta$ 8) $y = 4$

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