Simplifying Rational Expressions

A *rational expression* is a quotient of two monomials and/or polynomials.

Examples: $\dfrac{3x+1}{2}$, $\dfrac{x^2-1}{5x^2+2x}$, $\dfrac{3y}{5y^2}$, $\dfrac{1}{2}$, etc.

Beginning Steps:
1. Factor out the greatest common factor (**GCF**), if any, in all numerators and denominators.
2. **Factor completely** all numerators and denominators.
3. Remaining steps depend on the type of problem. See examples below for three types of problems: (I) single expression; (II) products and quotients; (III) sums and differences.

Type I: Single Rational Expression

Example #1 - Simplify: $\dfrac{w^2+w-12}{w^2+8w+16}$.

Solution - $\dfrac{w^2+w-12}{w^2+8w+16}$

1. there is no GCF in either numerator or denominator

$= \dfrac{(w-3)\,(w+4)}{(w+4)\,(w+4)}$

2. factor numerator and denominator completely

$= \boxed{\dfrac{(w-3)}{(w+4)}}$

3. cancel any like factors (representing 1)

Example #2 - Simplify: $\dfrac{3x+6}{3x}$.

Solution - $\dfrac{3x+6}{3x}$

$= \dfrac{3(x+2)}{3x}$

1. factor out GCF (of 3) in the numerator

$= \dfrac{3}{3} \cdot \dfrac{(x+2)}{x}$

2. there's no other factorization to be done

$= \boxed{\dfrac{x+2}{x}}$

3. cancel like factors (representing 1)

Note- it is **incorrect** to cancel as follows: $\dfrac{3x+6}{3x}$ 7 since the 3x in 3x+6 is not a factor but a term of the sum: 3x+6.

However, $\dfrac{3(x+2)}{3x} = \dfrac{x+2}{x}$ is correct.

Example #3 - Simplify: $\dfrac{150x-6x^3}{6x^2-27x-15}$.

Solution - $\dfrac{150x-6x^3}{6x^2-27x-15}$

$= \dfrac{6x(25-x^2)}{3(2x^2-9x-5)}$

1. factor out GCF of 6x in numerator and of 3 denominator

$= \dfrac{3\cdot2x(5+x)\,(5-x)}{3(2x+1)\,(x-5)}$

2. factor numerator and denominator completely

$= \dfrac{3\cdot2x\,(5+x)\,(-1)\,(x-5)}{3(2x+1)\,(x-5)}$

3. cancel like factors (representing 1)

$= \boxed{\dfrac{-2x(5+x)}{2x+1}}$

Note: $5-x = -1(-5+x) = -1(x-5)$. (factoring out -1 and rewriting)

Type II: Products and Quotients

Example #4- Simplify: $\dfrac{4x^2 - 9}{x + 1} \div \dfrac{10x^2 + 19x + 6}{x^2 + 8x + 7} \cdot \dfrac{5x + 10}{2x - 3}$.

Solution -

$\dfrac{4x^2 - 9}{x + 1} \div \dfrac{10x^2 + 19x + 6}{x^2 + 8x + 7} \cdot \dfrac{5x + 10}{2x - 3}$

$= \dfrac{4x^2 - 9}{x + 1} \cdot \dfrac{x^2 + 8x + 7}{10x^2 + 19x + 6} \cdot \dfrac{5x + 10}{2x - 3}$ (i) change division to mult. by the reciprocal

$= \dfrac{(2x + 3)\,(2x - 3)}{x + 1} \cdot \dfrac{(x + 7)\,(x + 1)}{(2x + 3)\,(5x + 2)} \cdot \dfrac{5(x + 2)}{2x - 3}$ (ii) factor completely and cancel

$= \boxed{\dfrac{5(x + 2)\,(x + 7)}{5x + 2}}$ (iii) write final answer as one fraction

Example #5 - Simplify: $\dfrac{3x - 1}{2x^2 - 9x - 5} \cdot \dfrac{x^2 + 2x - 8}{6x - 2}$.

Solution -

$\dfrac{3x - 1}{2x^2 - 9x - 5} \cdot \dfrac{x^2 + 2x - 8}{6x - 2}$

$= \dfrac{3x - 1}{(2x + 1)\,(x - 5)} \cdot \dfrac{(x + 4)\,(x - 2)}{2(3x - 1)}$ (i) factor completely

$= \boxed{\dfrac{(x + 4)\,(x - 2)}{2(2x + 1)\,(x - 5)}}$ (ii) cancel like factors

Type III: Sums and Differences

Example #6 - Add and simplify: $\dfrac{3y}{y^2 + 12y + 20} + \dfrac{y + 7}{y^2 - 100}$ (i) factor denominators & find LCD

Solution -

$\dfrac{3y}{(y+10)\,(y+2)} + \dfrac{y + 7}{(y + 10)\,(y - 10)}$ (ii) LCD is $(y +10)(y + 2)(y - 10)$

$= \dfrac{3y}{(y+10)\,(y+2)} \cdot \dfrac{(y - 10)}{(y - 10)} + \dfrac{y + 7}{(y + 10)\,(y - 10)} \cdot \dfrac{(y + 2)}{(y + 2)}$ (iii) write all fractions with LCD

$= \dfrac{3y^2 - 30y}{(y + 10)\,(y + 2)\,(y - 10)} + \dfrac{y^2 + 9y + 14}{(y + 10)\,(y + 2)\,(y - 10)}$ (iv) add fractions

$= \dfrac{3y^2 - 30y + y^2 + 9y + 14}{(y + 10)\,(y + 2)\,(y - 10)}$ (v) if possible, factor and reduce

$= \boxed{\dfrac{4y^2 - 21y + 14}{(y + 10)\,(y + 2)\,(y - 10)}}$ (vi) (cannot factor or reduce further)

Example #7 - Subtract and simplify: $\dfrac{x + 3}{2x^2 + 13x + 6} - \dfrac{5}{6x + 3}$

Solution -

$\dfrac{x + 3}{2x^2 + 13x + 6} - \dfrac{5}{6x + 3}$ (i) factor all denominators and find LCD

$= \dfrac{x + 3}{(2x + 1)\,(x + 6)} - \dfrac{5}{3(2x + 1)}$ (ii) LCD is: $3(2x + 1)(x + 6)$

$= \dfrac{x + 3}{(2x + 1)\,(x + 6)} \cdot \left(\dfrac{3}{3}\right) - \dfrac{5}{3(2x + 1)} \cdot \dfrac{(x + 6)}{(x + 6)}$ (iii) write all fractions with LCD

$= \dfrac{3x + 9}{3(2x + 1)\,(x + 6)} - \dfrac{5x + 30}{3(2x + 1)\,(x + 6)}$ (iv) subtract fractions

$= \dfrac{3x + 9 - 5x - 30}{3(2x + 1)\,(x + 6)}$ $= \boxed{\dfrac{-2x - 21}{3(2x + 1)\,(x + 6)}}$ (v) (cannot factor or reduce further)

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