WILLIAM D. LAW, JR. LEARNING COMMONS

Writing Equivalent Fractions

A.  $\frac{3}{4}$ of rectangle A is shaded.

B.  $\frac{6}{8}$ of rectangle B is shaded.

It is easy to see that the shaded parts of these rectangles are the same; therefore $\frac{3}{4}$ and $\frac{6}{8}$ are two ways of writing the same value. We say that $\frac{3}{4}$ and $\frac{6}{8}$ are **<u>equivalent fractions</u>**.

I. To **<u>build</u>** equivalent fractions means to write a fraction equivalent to the original fraction. The new fraction's numerator and denominator will be greater than those of the original fraction. We do this by using <u>The Multiplication Property of One</u> (Locate this property in your text).

$\frac{3}{4}\times 1 = \frac{3}{4}$ (This fraction has the same name as the original.)

We know that $\frac{2}{2} = 1$ (A natural number divided by itself is 1.)

$$\frac{3}{4}\times\frac{2}{2}=\frac{3\times 2}{4\times 2}=\frac{6}{8}$$

Since $\frac{3}{4}\times\frac{2}{2}$ is a way of showing $\frac{3}{4}\times 1$, we know the answer is equivalent to $\frac{3}{4}$.

Other fractions equivalent to $\frac{3}{4}$ can be found by multiplying $\frac{3}{4}$ by other names for 1.

1. a. $\frac{3}{4}\times\frac{3}{3}=$ b. $\frac{3}{4}\times\frac{5}{5}=$ c. $\frac{3}{4}\times\frac{10}{10}=$

Suppose you wanted to build a fraction equivalent to $\frac{3}{4}$ and you wanted its denominator to be 80.

$$\frac{3}{4}\times\frac{?}{?}=\frac{?}{80}$$

Look at the denominators. You are looking for a number to multiply by 4 to get 80. To determine the number, simply divide 80 by 4.

$$4\overline{)80}\quad 20$$

Now write a name for 1 using 20 in the denominator.

$$\frac{20}{20}=1 \text{ (The numerator must be the same as the denominator.)}$$

You can now find the fraction with a denominator of 80 that is equivalent to $\frac{3}{4}$

$$\frac{3}{4}\times\frac{20}{20}=\frac{60}{80}$$

Once you understand this procedure, you may omit the step which **shows** the name for 1.

1. divide the small denominator into the large denominator.
2. multiply the numerator by that quotient.

$$\frac{3}{4}=\frac{?}{80}$$

1. THINK: $4\overline{)80}\quad 20$

2. $3\cdot 20=60$

$$\frac{3}{4}=\frac{60}{80}$$

As you study the information on fractions, look for the uses of building equivalent fractions.

2-4. Write the fractions that are equivalent to the given fraction and have the indicated denominator.

2. $\frac{5}{8}=\frac{\quad}{72}$ 3. $\frac{1}{9}=\frac{\quad}{36}$ 4. $\frac{5}{12}=\frac{\quad}{156}$

5-7. Use the short-cut to write these equivalent fractions.

5. $\frac{5}{6}=\frac{\quad}{42}$ 6. $\frac{1}{7}=\frac{\quad}{84}$ 7. $\frac{3}{2}=\frac{\quad}{12}$

REMEMBER the numerator must change if the denominator changes!

II. **Simplest Form of Fractions**

When we write $\frac{6}{8}=\frac{3}{4}$, we are writing the value of the fraction in its simplest form. This is **sometimes** called reducing the fraction, although **simplifying** is a better description.

Look at the prime factorizations of 6 and 8.

$$\frac{6}{8}=\frac{2\times 3}{2\times 2\times 2}$$

We see a common factor of 2 in the numerator and denominator. Since $\frac{2}{2}=1$, we can write

$$\frac{6}{8}=\frac{2\times 3}{2\times 2\times 2}=\frac{2}{2}\times\frac{3}{2\times 2}=1\times\frac{3}{4}=\frac{3}{4}$$

A fraction is in its simplest form when the only common factor of the numerator and denominator is 1.

(Rhetorical question - isn't 1 always a common factor of two numbers?) When we say there is no common factor, we mean no factor other than 1 is in both numbers.

We should always write fraction answers in simplest form. This means
1. No common factor (other than 1) in the numerator and denominator.
2. Improper fractions should be written as mixed numbers.

NOTICE that either step can be done first.

Simplify: $\frac{12}{9}$

a. $\frac{12}{9}=\frac{3\times 4}{3\times 3}=\frac{4}{3}=1\frac{1}{3}$

or: b. $\frac{12}{9}=1\frac{3}{9}=1+\frac{3\times 1}{3\times 3}=1\frac{1}{3}$

The prime factorizations will help. Get rid of each pair of common factors.

$\frac{72}{90}=\frac{\cancel{2}\times\cancel{2}\times 2\times\cancel{3}\times\cancel{3}}{\cancel{2}\times 3\times\cancel{3}\times\cancel{5}}=\frac{4}{5}$ This works because $\frac{2}{2}=1$; $\frac{3}{3}=1$, etc. Each 1 is a factor.

8-12. Reduce each fraction to its simplest form.

8. $\frac{20}{30}$ 9. $\frac{16}{64}$ 10. $\frac{28}{16}$

11. $\frac{54}{18}$ 12. $\frac{180}{315}$

13. What does it mean to write a fraction in its simplest form?

ANSWERS:

1. a. $\frac{9}{12}$ b. $\frac{15}{20}$ c. $\frac{30}{40}$

2. $\frac{5}{9}\times\frac{9}{9}=\frac{45}{72}$

3. $\frac{1}{9}\times\frac{4}{4}=\frac{4}{36}$

4. $\frac{5}{12}\times\frac{13}{13}=\frac{65}{156}$

5. $\frac{5}{6}=\frac{35}{42}$

6. $\frac{1}{7}=\frac{12}{84}$

7. $\frac{3}{2}=\frac{18}{12}$ (NOTICE both values are greater than 1.)

8. $\frac{2}{3}$ 9. $\frac{1}{4}$ 10. $\frac{7}{4}=1\frac{3}{4}$ 11. $\frac{3}{1}=3$ 12. $\frac{4}{7}$

13. The only common factor of the numerator and denominator is 1. Improper fractions are written as mixed or whole numbers. (In algebra you will often leave the answer as an improper fraction.)