Solving General Inequalities

A general inequality is solved in the same way that a general equation is solved. The only difference is in the last step. If the coefficient of the variable term is negative we will be multiplying or dividing on both sides by a negative number. This means we will reverse the direction of the inequality. If the coefficient is positive the direction of the inequality will stay the same.

EXAMPLE: Solve: $3x + 2 > 14$

$3x + 2 + (-2) > 14 + (-2)$ Add (-2) to both sides.

$3x + 0 > 12$

coefficient of x is positive ---> $3x > 12$

$\downarrow$

$\frac{1}{3} \bullet 3x > 12 \bullet \frac{1}{3}$ Multiply both sides by $\frac{1}{3}$.

$\downarrow$

$\frac{3}{3}x > \frac{12}{3}$

$\downarrow$

$x > 4$

EXAMPLE: Solve: $-5x + 4 < 2x - 17$

$-5x + (-2x) + 4 < 2x + (-2x) - 17$ Add -2x to both sides.

$-7x + 4 < 0 - 17$

$-7x + 4 < -17$

$-7x + 4 + (-4) < -17 + (-4)$ Add -4 to both sides.

$-7x + 0 < -21$

coefficient of x is negative --> $-7x < -21$

$\downarrow$

$(-\frac{1}{7})(-7x) > (-21)(-\frac{1}{7})$ Multiply both sides by 1 and

$\downarrow$

$(\frac{-7}{-7})x > (\frac{-21}{-7})$ Reverse the inequality symbol.

$\downarrow$

$x > 3$

EXAMPLE: Solve**:** $2(5x - 8) < 7(x - 3)$ Use the Distributive Property

$10x - 16 < 7x - 21$

$10x - 7x - 16 < 7x - 21$ Subtract 7x from both sides-

$3x - 16 < 0 - 21$ (This is the same as adding -7x)

$3x - 16 < -21$

$3x - 16 + 16 < -21 + 16$ Add 16 to both sides

$3x + 0 < -5$

Coefficient of x is positive --> $3x < -5$

$\frac{1}{3} \bullet 3x < -5 \bullet \frac{1}{3}$ <-------Multiply both sides by $\frac{1}{3}$.

$\downarrow$

$\frac{3}{3}x < \frac{-5}{3}$ The symbol stays the same.

$\downarrow$

$x < \frac{5}{3}$

EXAMPLE: Solve: $5(2 - x) > 3(2x - 5)$ Use the Distributive Property

$10 - 5 > 6x - 15$

$10 - 5x - 6x > 6x - 6x - 15$ Subtract 6x from both sides

$10 - 11x > 0 - 15$

$10 - 11x > -15$

$10 - 10 - 11x > -15 - 10$ Subtract 10 from both sides

$0 - 11x > -25$

Coefficient of x is negative--> $-11x > -25$

$\downarrow$

$\left(-\frac{1}{11}\right)(-11x) < (-25)\left(-\frac{1}{11}\right)$ <--Multiply both sides by $-\frac{1}{11}$

$\downarrow$ and reverse the inequality symbol.

$\left(\frac{-11}{-11}\right)x < \left(\frac{-25}{-11}\right)$

$\downarrow$

$x < \frac{25}{11}$

EXERCISES:

1. $7x - 3 \geq 6x - 2$
2. $10 - 3y \leq 7y$
3. $2x - 9 \geq 5x + 4$
4. $8y - 9 > 3y - 9$
5. $4(3x - 1) > 3(2 - 5x)$
6. $3(2x - 5) > 8x - 5$
7. $3x - 2(3x - 5) \geq 4(2x - 1)$
8. $15 - 5(3 - 2x) \leq 4(x - 3)$

KEY:

1. $x \geq 1$
2. $y \geq 1$
3. $x \leq -\frac{13}{3}$
4. $y > 0$
5. $x > \frac{10}{27}$
6. $x < -5$
7. $x \leq \frac{14}{11}$
8. $x \leq -2$