Converting Among Fractions, Decimals, and Percents

Converting Between Fractions and Decimals

The set of rational numbers includes all fractions and integers. Each of the fractions can be written in decimal form.

> To convert from a fraction to a decimal, divide the numerator by the denominator. The decimal value of every rational number will either terminate or it will repeat. We place a bar over the digits that repeat.

EXAMPLES

a. $\dfrac{5}{8}$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ 4\,8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

or $\qquad 5 \div 8 = 0.625$

Because the remainder is 0, we say the decimal *terminated*.

b. $\dfrac{5}{6}$

$$\begin{array}{r} 0.833 \\ 6\overline{)5.000} \\ 4\,8 \\ \hline 20 \\ 18 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

Because the quotient repeats the 3, we say this is a *repeating decimal*.

We write $0.8\overline{3}$ where the bar shows the repeating part.

Remember that $\dfrac{3}{8}$ is less than one, so its decimal value will be less than one.

$$\frac{3}{8} = 0.375$$

The value of $\dfrac{8}{3}$ is greater than one since the numerator is larger than the denominator.

$$\frac{8}{3} = 2.6666\ldots = 2.\overline{6}$$

We can check whether the decimal equivalent of a fraction is reasonable by keeping in mind the value of the fraction. If you divide by the wrong number, your answer will not be reasonable.

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You will hear more about rational, irrational, and real numbers in this and future math courses. For now, you should know that all of the numbers on the number line are the real numbers. The real numbers that can be written as a ratio of two integers are rational numbers—their decimal values will either terminate or repeat. The real numbers that cannot be written in fraction form are irrational numbers. Their decimal values do not terminate and they do not repeat.

> To convert a terminating decimal to a fraction, read the decimal and write the corresponding fraction. Reduce the fraction to lowest terms.

EXAMPLES

a. 0.36 is read "thirty-six hundredths," which is written as the fraction $\frac{36}{100}$. Reduce.

$$\frac{36}{100} = \frac{2 \cdot 2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 5 \cdot 5} = \frac{9}{25}$$

b. 0.007 is read "seven thousandths," or $\frac{7}{1000}$. This fraction is already in lowest terms.

Converting Between Fractions, Decimals, and Percents

The word percent means "parts of 100." There are several different ways we can say "parts of 100."

- The fraction $\frac{3}{100}$ means 3 parts of 100.
- The decimal 0.03 also means 3 parts of 100.
- The percent 3% means 3 parts of 100. We use the % symbol to mean $\frac{1}{100}$.

In solving percent problems we must be able to convert from the percent form to the equivalent decimal or fraction.

> To write a percent as a fraction:
>
> 1. Drop the percent sign
> 2. Multiply by $\frac{1}{100}$

EXAMPLE: Write 86% as a fraction.

$$86\% = 86 \times \frac{1}{100} = \frac{86}{100} = \frac{43}{50}$$

EXAMPLE: Write $5\frac{1}{4}\%$ as a fraction.

$$5\frac{1}{4}\% = 5\frac{1}{4} \times \frac{1}{100} = \frac{21}{4} \times \frac{1}{100} = \frac{21}{400}$$

EXAMPLE: Write $\frac{5}{8}\%$ as a fraction.

$$\frac{5}{8}\% = \frac{5}{8} \times \frac{1}{100} = \frac{\cancel{5}}{8} \times \frac{1}{\cancel{5} \cdot 20} = \frac{1}{160}$$

To write a percent as a decimal:

1. Drop the percent sign
2. Multiply by 0.01. This moves the decimal point two places to the left.

EXAMPLE: Write 170% as a decimal.

$$170\% = 170(0.01) = 1.70 = 1.7$$

EXAMPLE: Write 3% as a decimal.

$$3\% = 3(0.01) = 0.03$$

EXAMPLE: Write 12.7% as a decimal.

$$12.7\% = 12.7(0.01) = 0.127$$

To write a decimal as a percent:

1. Multiply by 100. This moves the decimal point two places to the right.
2. Attach the percent sign.

EXAMPLE: Write 0.024 as a percent.

$$0.024(100\%) = 2.4\%$$

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EXAMPLE: Write 3.15 as a percent.

$$3.15(100\%) = 315\%$$

EXAMPLE: Write 0.0027 as a percent.

$$0.0027(100\%) = 0.27\%$$

To write a fraction as a percent:

   1. Multiply by 100.
   2. Attach the percent sign.

EXAMPLE: Write $\dfrac{7}{20}$ as a percent.

$$\frac{7}{20} \times 100\% = \frac{7}{\cancel{20}_{1}} \cdot \frac{\cancel{100}^{5}}{1}\% = 35\%$$

EXAMPLE: Write $3\dfrac{2}{5}$ as a percent.

$$3\frac{2}{5} \times 100\% = \frac{17}{\cancel{5}_{1}} \cdot \frac{\cancel{100}^{20}}{1}\% = 340\%$$

EXAMPLE: Write $\dfrac{5}{11}$ as a percent. Round to the nearest tenth of a percent.

$$\frac{5}{11} \times 100\% = \frac{5}{11} \cdot \frac{100}{1}\% = \frac{500}{11}\%$$

Divide to convert to a decimal.

$$11\overline{)500.000}$$
$$\begin{array}{r} 45.454 \\ \underline{44} \\ 60 \\ \underline{55} \\ 50 \\ \underline{44} \\ 60 \end{array}$$

This is a repeating decimal. $\dfrac{5}{11} = 45.5\%$ to the nearest tenth.

EXAMPLE: Write $\dfrac{5}{6}$ as a percent. Write the remainder in fractional form (use a mixed number).

$$\frac{5}{6}\times100\% = \frac{5}{\cancel{6}_{3}}\cdot\frac{\overset{50}{\cancel{100}}}{1}\% = \frac{250}{3}\%$$

Divide

$$3\overline{)250}$$
$$\underline{24}$$
$$10$$
$$\underline{9}$$
$$1$$

$$\frac{5}{6} = \frac{250}{3}\% = 83\frac{1}{3}\%$$

EXERCISES:

Write as a fraction:

1. 25%   2. 450%   3. $6\dfrac{1}{4}\%$

4. $\dfrac{1}{8}\%$ 5. $33\dfrac{1}{3}\%$

Write as a decimal:

6. 600%   7. 0.27%   8. 38%

9. 1.296%   10. 13%

Write as a percent. Write any remainders in fractional form.

11. 1.35   12. 0.003   13. $\dfrac{3}{8}$

14. $\dfrac{9}{20}$ 15. $\dfrac{5}{16}$

Write as a percent. Round to the nearest tenth of a percent if necessary.

16. 3.92   17. 5   18. $\dfrac{2}{3}$

19. $\dfrac{5}{8}$ 20. $\dfrac{5}{9}$

Write as a decimal. If the decimal repeats, use a bar over the repeating digits.

21. $\dfrac{1}{8}$ 22. $\dfrac{3}{16}$ 23. $\dfrac{3}{11}$ 24. $\dfrac{7}{110}$

KEY:

1. $\dfrac{1}{4}$ 2. $4\dfrac{1}{2}$ 3. $\dfrac{1}{16}$ 4. $\dfrac{1}{800}$ 5. $\dfrac{1}{3}$ 6. 6

7. 0.0027   8. 0.38   9. 0.01296   10. 0.13   11. 135%   12. 0.3%

13. $37\dfrac{1}{2}$%   14. 45%   15. $31\dfrac{1}{4}$%   16. 392%   17. 500%   18. 66.7%

19. 62.5%   20. 55.6%   21. 0.125   22. 0.1875   23. $0.\overline{27}$ 24. $0.0\overline{63}$