Proportions

A **ratio** is a comparison of two quantities with the same unit of measure. The ratio of 24 ounces to 18 ounces is written as:

$$\frac{24}{18} = \frac{4}{3}$$ (Note: The units of measure have cancelled since they are identical.)

A **rate** is a comparison of two quantities with different units of measure. The comparison of 53 miles driven to 2 gallons of gasoline used is the rate:

$$\frac{53\,\text{miles}}{2\,\text{gallons}}$$ (Note: The units of measure appear in the answer.)

Never write a ratio or a rate as a whole number or a mixed number.

A **proportion** is the statement that two ratios or two rates are equal. A proportion is true if the cross-products are equal.

Example: $\frac{120}{4} = \frac{150}{5}$ The cross-products are $5 \cdot 120 = 600$ and $4 \cdot 150 = 600$. The proportion is true because the cross-products are

If we know three of the four quantities in a proportion, we can solve for the unknown quantity. Let's solve the following proportion:

$\frac{7}{n} = \frac{12}{8}$ Write the cross-products and show that they are equal.

$$7 \cdot 8 = 12n$$
$$56 = 12n$$
$$\frac{56}{12} = \frac{12n}{12}$$
$$\frac{14}{3} = n$$

If we divide, we get $n = 4.6666...$, and if we round to the nearest tenth, we will have an approximate value of $n \approx 4.7$.

Proportions are useful in many "real life" situations. For example, Linda needs to prepare a casserole for 21 people. Her recipe gives the ingredients to serve 6 people. If the recipe calls for 8 ounces (oz.) of cream cheese, how many ounces will Linda need?

To solve this problem, write the rate of ounces to people given in the recipe, then write the rate of ounces to people that she needs using the variable *n* for the unknown number of ounces:

$$\frac{8\,\text{oz}}{6\,\text{people}} = \frac{n\,\text{oz}}{21\,\text{people}}$$

Solve this proportion by cross multiplying (It is easier to do the math without writing the units of measure; however, do not forget to put the correct unit of measure on the final answer.):

$$\frac{8}{6} = \frac{n}{21}$$
$$8 \cdot 21 = 6n$$
$$168 = 6n$$
$$\frac{168}{6} = \frac{6n}{6}$$
$$n = 28\,\text{oz of cream cheese}$$

The original proportion could have been written with the number of people to ounces as follows:

$$\frac{6\,\text{people}}{8\,\text{oz}} = \frac{21\,\text{people}}{n\,\text{oz}}$$

The cross products will give us $6n = 8 \cdot 21$, which gives the same answer as above.

It is important that the numbers in the two fractions have the same label in the same position. Let both numerators represent the same units and then both denominators represent the same units. If "people" are in the first numerator, "people" must be in the second numerator. If ounces are in the first denominator, then ounces must be in the second denominator.

You may also solve a proportion by multiplying each term by the Least Common Denominator (LCD) as we did when we solved other equations containing fractions:

$\frac{8}{6} = \frac{n}{21}$ Now multiply each term by the LCD, which is 42.

$$42\left(\frac{8}{6}\right) = \left(\frac{n}{21}\right)42$$
$$56 = 2n$$
$$\frac{56}{2} = \frac{2n}{2}$$
$$28 = n$$

Example: Proportions are used in the nursing profession. In estimating a medication dosage for a child, the ratio of the body weight of the child to that of an average adult (150 pounds) is used. If an average adult (150 pounds) receives 50 mg of Benadryl, how much Benadryl would a child weighing 27 pounds receive?

Let x = the number of mg of Benadryl needed for the child.

$$\frac{50\,\text{mg}}{150\,\text{lb}} = \frac{x\,\text{mg}}{27\,\text{lb}}$$

$$\frac{50}{150} = \frac{x}{27}$$

$$50 \cdot 27 = 150x$$

$$1350 = 150x$$

$$\frac{1350}{150} = \frac{150x}{150}$$

$$x = 9$$

Therefore, the 27 pound child would need a dosage of 9 mg of Benadryl.

Exercises: In 1–4, find the cross-products. If they are equal, write **TRUE**. If they are not equal, write **NOT TRUE**.

1. $\frac{66\,\text{in.}}{21\,\text{in.}} = \frac{110\,\text{in.}}{35\,\text{in.}}$

2. $\frac{\$.84}{9\,\text{oz.}} = \frac{\$.19}{2\,\text{oz.}}$

3. $\frac{\$21}{6\,\text{hrs.}} = \frac{\$35}{10\,\text{hrs.}}$

4. $\frac{562\text{ mi.}}{9\,\text{hrs.}} = \frac{378\,\text{mi.}}{6\,\text{hrs.}}$

In 5-7, solve the following proportions:

5. $\frac{n}{12} = \frac{7}{3}$

6. $\frac{8}{15} = \frac{n}{9}$

7. $\frac{10}{n} = \frac{8}{100}$

8. $\frac{9}{20} = \frac{6}{n}$

9. Two hundred people were surveyed. One hundred eighty of these surveyed were willing to pay more taxes to help the homeless. At this rate, how many people in a town of 15,700 people would be willing to pay more taxes? (**At this rate** implies there are two rates that are the same; that tells you to write a proportion.)

10. Three tablespoons of fertilizer must be added to 4 gallons of water. How many gallons of water should be mixed with 1 tablespoon of fertilizer? (Write your answer as a mixed number.)

11. A dosage of medicine is 2 mg for every 50 lbs. of body weight. How many mg of medicine should a person who weighs 125 lbs. take?

12. Sam drove his car 400 miles on 15 gallons of gas. How many gallons should it take him to drive 640 miles?

13. A dosage of medicine for an adult weighing 150 pounds is 100 mg. How much of this medication would be given to a child weighing 60 pounds?

14. If 2 pounds of fertilizer will cover 75 square feet of garden, to the nearest tenth of a pound, how many pounds would be needed for 325 square feet?

15. The taxes on a $60,000 house are $850. At this same tax rate, what would the taxes be on a $110,000 house? Give the answer to the nearest cent.

16. Eighteen yards of material are needed for five dresses. How much material is needed for twelve dresses?

17. A garden service charges $40 to install 50 square feet of sod. Find the charge to install 225 square feet of sod.

18. On a road map, 2 inches represents 6 miles. How many inches, to the nearest tenth of an inch, would represent a distance of 40 miles?

Answer Key:

1. True	2. Not True	3. True
4. Not True	5. 28	6. 4.8
7. 125	8. 13.3	9. 14,130 people
10. $1\frac{1}{3}$ gallons	11. 5 mg	12. 24 gallons
13. 40 mg	14. 8.7 pounds	15. $ 1558.33
16. 43.2 yards	17. $ 180.00	18. 13.3 inches

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