

The Quadratic Formula

Using the **quadratic formula**, we can solve all quadratic equations.

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve the equations $6x - 1 = x^2$

First we put the equation in **standard form** by subtracting x^2 from each side.

$$-x^2 + 6x - 1 = 0$$

We will use the **quadratic formula**: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = -1$, $b = 6$, $c = -1$.

$$\frac{-6 \pm \sqrt{(6)^2 - (4)(-1)(-1)}}{2(-1)}$$

Substitute $a = -1$, $b = 6$, $c = -1$ into the formula. Place the parentheses on the numbers to avoid making mistakes on “signs”

$$\frac{-6 \pm \sqrt{36 - 4}}{-2}$$

Simplify.

$$\frac{-6 \pm \sqrt{32}}{-2}$$

Simplify the radical part, using the fact that $\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$.

$$\frac{-6 \pm 4\sqrt{2}}{-2} \quad \text{or} \quad \frac{-6}{-2} \pm \frac{4\sqrt{2}}{-2}$$

Factor the numerator (-2 is a factor of both terms in the numerator).

$$\frac{-2(3 \pm 2\sqrt{2})}{-2}$$

Cancel the common factor of -2 from the numerator and denominator.

$$3 \pm 2\sqrt{2}$$

There are two distinct solutions.

$$3 + 2\sqrt{2} \quad \text{and} \quad 3 - 2\sqrt{2}$$

Note: the fact that $b^2 - 4ac$ is not equal to a perfect square indicates that it is not possible to solve this equation by factoring.

Exercises: Solve the equations using quadratic formula.

1. $x^2 + 2x - 24 = 0$ 2. $2x(x - 3) = 2$ 3. $\frac{1}{2}x^2 + \frac{3}{2}x - 2 = 0$ 4. $7x^2 + 4 = 2x$

Answers:

1. $\{4, -6\}$ 2. $\left\{\frac{3 \pm \sqrt{13}}{2}\right\}$ 3. $\{-4, 1\}$ 4. $\left\{\frac{1 \pm 3i\sqrt{3}}{7}\right\}$