

Systems of Linear Equations

EXAMPLE

1. Solve for the system of linear equations.

$$\begin{aligned} 4x + 2y &= -6 \\ 5x + 5y &= 10 \end{aligned}$$

SOLUTION

$$\begin{aligned} 4x + 2y &= -6 \\ 5x + 5y &= 10 \end{aligned}$$

We will eliminate y .

Multiply the first equation by -5 and multiply the second equation by 2 .

$$\begin{array}{r} -20x - 10y = 30 \\ \underline{10x + 10y = 20} \\ -10x \qquad = 50 \end{array}$$

Add the two equations.

$$-10x = 50$$

Solve for x .

$$x = -5$$

Let $x = -5$ in $4x + 2y = -6$.

$$4(-5) + 2y = -6$$

$$-20 + 2y = -6$$

Solve for y .

$$2y = 14$$

$$y = 7$$

The solution occurs when $x = -5$, $y = 7$.

The solution is $(-5, 7)$

The system is **consistent** and **independent** system.

EXAMPLE

2. Solve for the system of linear equations.

$$12x - 4y = 20$$

$$9x - 3y = 5$$

SOLUTION

$$12x - 4y = 20$$

$$9x - 3y = 5$$

$$-36x + 12y = -60$$

$$\underline{36x - 12y = 20}$$

$$0 = -40$$

We will eliminate y .

Multiply the first equation by -3 and multiply the second equation by 4 .

Add the two equations.

Since use of the addition method has resulted in the case where all of the variables cancel and we are left with a false statement ($0 = -40$), we know that the system of equations has no solution.

This is an **inconsistent** system.

EXAMPLE

3. Solve for the system of linear equations

$$\begin{aligned}x - 5y &= 3 \\ -4x + 20y &= -12\end{aligned}$$

SOLUTION

$$\begin{aligned}x - 5y &= 3 \\ -4x + 20y &= -12\end{aligned}$$

$$\begin{aligned}4x - 20y &= 12 \\ \underline{-4x + 20y} &= \underline{-12}\end{aligned}$$

$$0 = 0$$

We will eliminate x .

Multiply the first equation by 4.

Add the two equations.

Since use of the addition method has resulted in the case where all of the terms cancel on both sides of the equals sign, leaving the true statement $0 = 0$, we know that there are infinitely many solutions.

$$\{(x, y) \mid x - 5y = 3\}$$

This is a **consistent** and **dependent** system.

Exercises: Solve for the following systems of linear equations

1. $3x + 2y = -1$
 $2x - 3y = -5$

2. $5x - 2y = 5$
 $-10x + 4y = -8$

3. $2x - 4y = -10$
 $5x + 4y = 17$

4. $5x - 2y = 5$
 $-10x + 4y = -10$

5. $3x - 4y = 5$
 $-3x + 4y = -3$

Answers:

1. $(-1, 1)$ 2. \emptyset 3. $(1, 3)$ 4. $\{(x, y) \mid 5x - 2y = 5\}$ 5. \emptyset