

CLAST SKILL II.E.2

The student will determine equivalence or non-equivalence of statements.

You will be given a simple or compound statement expressed in words, and will be asked to select an equivalent (or non-equivalent) statement. The statement will be a conjunction, disjunction, or conditional statement.

These problems may refer to any of the following rules of logical equivalency.

$$p \rightarrow q \equiv \sim p \vee q$$

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

$$\sim(\sim p) \equiv p$$

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\sim(p \rightarrow q) \equiv p \wedge \sim q$$

Notice that many of these rules are the same rules for negations that are tested under Skill II.E.1.

EXAMPLE A

Select the statement that is logically equivalent to “If I’m expecting visitors then I wash the dishes.”

- A. If I wash the dishes, then I’m expecting visitors.
- B. I wash the dishes, or I’m expecting visitors.
- C. If I don’t wash the dishes, then I’m not expecting visitors.
- D. If I’m not expecting visitors, then I don’t wash the dishes.

SOLUTION

Let p represent the statement “I’m expecting visitors.”

Let q represent the statement “I wash the dishes.”

The statement “If I’m expecting visitors then I wash the dishes” is symbolized $p \rightarrow q$.

From the facts outlined above, we see that there are two rules of logical equivalency that could be used.

1. $p \rightarrow q \equiv \sim p \vee q$

Applying this rule, we have the following equivalent statement: “I’m not expecting visitors or I wash the dishes.” This is not among the listed choices.

2. $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Applying this rule, we have the following equivalent statement: “If I don’t wash the dishes, then I’m not expecting visitors.” This is choice C.

The correct choice is C.

EXAMPLE B

Select the statement that is logically equivalent to “It is not true that either Fluffy or Whiskers are puppies.”

- A. Fluffy is a puppy and Whiskers is a puppy.
- B. Fluffy is not a puppy and Whiskers is not a puppy.
- C. Fluffy is not a puppy or Whiskers is not a puppy.
- D. If Fluffy is not a puppy, then Whiskers is not a puppy.

EXAMPLE B SOLUTION

Let p be the statement “Fluffy is a puppy.”

Let q be the statement “Whiskers is a puppy.”

The statement “It is not true that either Fluffy or Whiskers are puppies” is the negation of $p \vee q$, so we are asked to find the statement that is equivalent to the *negation* of $p \vee q$.

According to one of DeMorgan’s Laws, $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

In words, the statement $\sim p \wedge \sim q$ is “Fluffy is not a puppy and Whiskers is not a puppy.”

The correct choice is B.

EXAMPLE C

Select the statement that is not logically equivalent to “If you own a car, then you need insurance.”

- A. If you don’t own a car, then you don’t need insurance.
- B. You don’t own a car or you need insurance.
- C. If you don’t need insurance, then you don’t own a car.
- D. You need insurance or you don’t own a car.

EXAMPLE C SOLUTION

Let p be the statement “You own a car.”

Let q be the statement “You need insurance.”

The statement “If you own a car, then you need insurance” is symbolized as $p \rightarrow q$.

Since we are asked to choose the statement that is *not* equivalent to the given statements, three of the four answers must be equivalent to $p \rightarrow q$.

Using the rules of equivalency listed above, we have the following:

1. $p \rightarrow q \equiv \sim p \vee q$ Applying this rule to the given statement, we have this equivalent statement: “You don’t own a car or you need insurance.” This is choice B.
2. $p \rightarrow q \equiv \sim q \rightarrow \sim p$ Applying this rule to the given statement, we have the following equivalent statement: “If I don’t need insurance, then you don’t own a car.” This is choice C.

We need to find a third statement that is equivalent to the given conditional statement, but we have already exhausted both of the rules for equivalency for conditional statements. However, because the disjunction is symmetric, we can take choice B, “You don’t own a car or you need insurance,” and reverse its two components to get this equivalent statement: “You need insurance or you don’t own a car.” This is choice D.

We were asked to find the statement that was *not* equivalent to the given statement. We have determined that choices B, C and D *are* equivalent to the given statement, so the only remaining choice is A.

The correct choice (the non-equivalent statement) is A.

Exercises

1. Select the statement that is logically equivalent to "If I will retire in comfort, then I will start planning today."
 - A. If I won't retire in comfort, then I won't start planning today.
 - B. I will retire in comfort and I won't start planning today.
 - C. If I will start planning today, then I will retire in comfort.
 - D. If I won't start planning today, then I won't retire in comfort.

2. Select the statement that is logically equivalent to "It is not true that if you pass this course then you will graduate."
 - A. You pass this course and you won't graduate.
 - B. You won't pass this course or you will graduate.
 - C. If you don't pass this course then you won't graduate.
 - D. If you don't pass this course then you will graduate.

3. Select the statement that is logically equivalent to "If you aren't a careful driver, then you'll wreck your car."
 - A. If you are a careful driver, then you won't wreck your car.
 - B. If you wreck your car, then you aren't a careful driver.
 - C. You are a careful driver or you wreck your car.
 - D. You are a careful driver and you don't wreck your car.

4. Select the statement that is **not** logically equivalent to this statement (from the instructions for IRS Form 1040): "If you have paid too much, we will send you a refund."
 - A. If we won't send you a refund, then you haven't paid too much.
 - B. If you haven't paid too much, then we won't send you a refund.
 - C. You haven't paid too much or we will send you a refund.
 - D. We will send you a refund, or you haven't paid too much.

5. Select the statement that is logically equivalent to "It is not true that both roses are red and violets are blue."
 - A. Roses are red and violets aren't blue.
 - B. Roses are red or violets aren't blue.
 - C. Roses aren't red and violets aren't blue.
 - D. Roses aren't red or violets aren't blue.

6. Select the statement that is **not** logically equivalent to "If you aren't gullible, then you won't vote for me."

- A. If you won't vote for me, then you aren't gullible.
- B. If you will vote for me, then you are gullible.
- C. You won't vote for me, or you are gullible.
- D. You are gullible, or you won't vote for me.

7. Select the statement that is logically equivalent to "It is not the case that if I don't get a better job then I don't get a better apartment."

- A. If I get a better job, then I get a better apartment.
- B. If I don't get a better job, then I get a better apartment.
- C. I get a better job and I don't get a better apartment.
- D. I don't get a better job and I get a better apartment.

8. Select the statement that is logically equivalent to "If you know the answer, then you don't guess."

- A. You don't know the answer or you don't guess.
- B. You know the answer and you guess.
- C. If you don't know the answer, then you guess.
- D. If you don't guess, then you know the answer.

Answers

1. D 2. A 3. C 4. B 5. D 6. A 7. D 8. A