WILLIAM D. LAW, JR.
LEARNING **COMMONS**

Consecutive Integers

The set of integers is: $\{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

The word "consecutive" means one right after the other. Three consecutive integers starting with 7 would be 7, 8 and 9.

Three consecutive odd integers beginning with 7 would be 7, 9, and 11. Three consecutive even integers beginning 8 would be 8, 10, and 12.

A. Please respond to each of the following:

1. Name three consecutive integers beginning with 14:

2. Name three consecutive odd integers beginning with 15:

3. Name three consecutive even integers beginning with 16:

4. Name three consecutive integers beginning with –3:

5. Name three consecutive odd integers beginning with –5:

6. Name three consecutive even integers beginning with –6:

Graph the answers to the six questions above on a number line:

1. Graph #1:
4. Graph #4:
2. Graph #2:
5. Graph #5:
3. Graph #3:
6. Graph #6:

If you compare the graphs, you will discover that the three dots in the consecutive integer graphs (#1 and #4) are ONE unit apart. The three dots in both the consecutive odd integer graphs (#2 and #5) and the consecutive even integer graphs (#3 and #6) are TWO apart. That tells us how we can represent these integers in terms of a variable when we do not know what the integers are.

To represent consecutive integers, let the first integer be n. Then, since consecutive integers are one apart, to get the next consecutive integer, add "1" to n. The second integer will be $n + 1$. To get a third consecutive integer, add one to the previous integer and you will get $n + 1 + 1$, which when simplified, becomes $n + 2$.

To represent consecutive odd integers, let the first integer be n. Then since consecutive odd integers are two apart, to get the next consecutive odd integer, add two to n. The second integer will be $n + 2$. To get a third consecutive odd integer, add two to the previous integer and you will get $n + 2 + 2$, which, when simplified, becomes $n + 4$.

To represent consecutive even integers, let the first integer be n. Then add 2 for each consecutive even integer in the same manner that you used for consecutive odd integers.

To summarize:

1. two consecutive integers:
   n
   $n + 1$

2. three consecutive integers:
   n
   $n + 1$
   $n + 2$

3. two consecutive odd integers:
   n
   $n + 2$

4. three consecutive odd integers:
   n
   $n + 2$
   $n + 4$

5. two consecutive even integers:
   n
   $n + 2$

6. three consecutive even integers:
   n
   $n + 2$
   $n + 4$

Please notice that consecutive odd and consecutive even integers are set up the same way. If you look at your number lines, you will realize that this makes sense. The dots you placed on those number lines are TWO apart. If you have an odd integer and you want the next consecutive odd integer, you have to add two to it. If you start with an even integer and you want the next consecutive even integer, you also have to add two to it.

Now let us see how all of this applies to consecutive integer problems.

If the sum of two consecutive integers is 35, and you are trying to write an equation that will help you find those integers, the first thing to do is to represent the two integers in terms of a variable.

Let n be the first integer and $n + 1$ be the second integer

Then, to represent "the sum of two consecutive integers is 35" you would write the equation:

$$n + (n + 1) = 35$$

The parentheses were put there just to help you read it—they are not necessary. Now, solve this equation:

$$\begin{aligned} n + (n + 1) &= 35 \\ 2n + 1 &= 35 \\ \underline{\quad -1} &\ \underline{\ -1} \\ 2n &= 34 \\ n &= 17 \\ n + 1 &= 18 \end{aligned}$$

Therefore, the two consecutive integers whose sum is 35 are 17 and 18. To check these answers, be sure that the numbers are consecutive integers (17 and 18) and that their sum is 35 (17 + 18 = 35).

EXAMPLE 1: The sum of three consecutive odd integers is 51. Find the integers.

Let x be the first odd integer
$x + 2$ be the second odd integer and
$x + 4$ be the third odd integer.

Finding the sum of the three integers gives us the equation

$$x + (x + 2) + (x + 4) = 51$$

The parentheses are there only for readability.

SOLVING:

$$
\begin{aligned}
x + x + 2 + x + 4 &= 51 \\
3x + 6 &= 51 \\
\underline{\quad -6} &\underline{\;\; -6} \\
3x &= 45 \\
\\
x &= 15 \\
x + 2 &= 17 \\
x + 4 &= 19
\end{aligned}
$$

CHECK: Are these **<u>consecutive odd</u>** integers? Is their sum 51?

EXAMPLE 2: Find two consecutive even integers such that four times the first is equal to three times the second.

n = first even integer
$n + 2$ = second even integer

4 times the first is 3 times the second

$$
\begin{aligned}
4n &= 3(n + 2) \\
4n &= 3n + 6 \\
\underline{-3n} &\underline{\;\; -3n \qquad} \\
n &= 6 \\
n + 2 &= 8
\end{aligned}
$$

CHECK: Are these **<u>consecutive even</u>** integers? Is four times the first (smaller) the same as three times the second (larger)?

EXAMPLE 3: Find three consecutive integers such that three times the smallest is twenty-two more than the largest.

n = first integer
$n + 1$ = second integer
$n + 2$ = third integer

3 times the smallest is 22 more than largest

$$
\begin{aligned}
3n &= 22 + (n + 2) \\
3n &= 24 + n \\
\underline{-n} &\underline{\qquad -n \quad} \\
2n &= 24
\end{aligned}
$$

Notice that only the first (smallest) and (third) integers are used in this equation, but the answer includes all three of the integers.

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$$n = 12$$
$$n + 1 = 13$$
$$n + 2 = 14$$

CHECK: Are these **consecutive** integers? Is three times the smallest equal to twenty-two more than the largest?

EXAMPLE 4: Find three consecutive even integers such that three times the middle one is 20 more than the sum of the first and third.

n = first even integer
$n + 2$ = second even integer
$n + 4$ = third even integer

$$\begin{aligned}
3(n + 2) &= 20 + n + (n + 4) \\
3n + 6 &= 24 + 2n \\
-2n \quad & \quad -2n \\
n + 6 &= 24 \\
-6 \quad & \quad -6 \\
n &= 18 \\
n + 2 &= 20 \\
n + 4 &= 22
\end{aligned}$$

CHECK: Are these **consecutive even** integers? Is three times the middle one equal to twenty more than the sum of the first and third?

EXAMPLE 5: Find three consecutive odd integers such that twice the sum of the first and last integers is nine less than five times the middle one.

n = first odd integer
$n + 2$ = second odd integer
$n + 4$ = third odd integer

$$\begin{aligned}
2\left[n + (n + 4)\right] &= 5(n + 2) - 9 \\
2(2n + 4) &= 5n + 10 - 9 \\
4n + 8 &= 5n + 1 \\
-n + 8 &= 1 \\
-n &= -7 \\
n &= 7 \\
n + 2 &= 9 \\
n + 4 &= 11
\end{aligned}$$

CHECK: Are these **consecutive odd** integers? Is twice the sum of the first and last integers equal to nine less than 5 times the middle one?

EXERCISES Now try these. Write an equation and solve.

1. The sum of three consecutive integers is 18. Find the integers.

2. The sum of three consecutive even integers is 36. Find the integers.

3. Find two consecutive odd integers such that three times the larger is one less the four times the smaller.

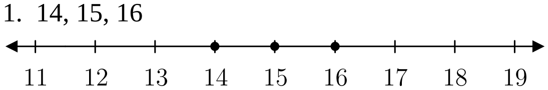
4. Find three consecutive integers such that four times the smallest one is nineteen more than the sum of the middle and largest ones.

5. Find three consecutive odd integers such that three times the largest is 13 less than twice the sum of the first two.

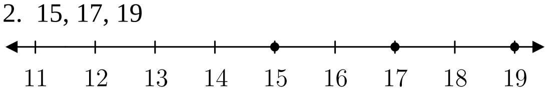
<u>ANSWERS:</u>

Part A.

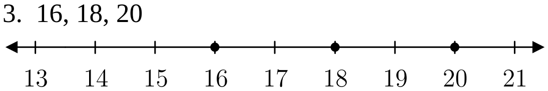
1. 14, 15, 16



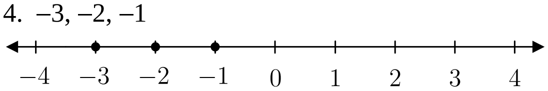
2. 15, 17, 19



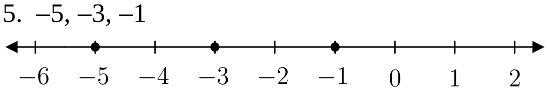
3. 16, 18, 20



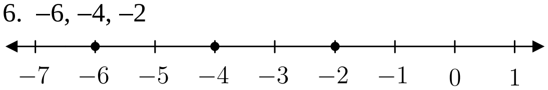
4. –3, –2, –1



5. –5, –3, –1



6. –6, –4, –2



Exercises

1. 5, 6, 7
2. 10, 12, 14
3. 7, 9
4. 11, 12, 13
5. 21, 23, 25

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