MONOMIAL FACTORS

Factoring means that we will be starting with a product and deciding what was multiplied together in order to get that product. To factor polynomials, we will first want to see whether there are any common factors.

Let us look at some examples with numbers.
The number 24 could be factored in several ways:

$24 = 1 \times 24$

$24 = 2 \times 12$

$24 = 3 \times 8$

$24 = 4 \times 6$

It looks like there are several answers to this problem. However, if we break each number down so that it is a product of prime factors we will see that there is really only one set of factors.

$24 = 1 \times 24$	$1 \times 2 \times 2 \times 2 \times 3$
$24 = 2 \times 12$	$2 \times 2 \times 2 \times 3$
$24 = 3 \times 8$	$3 \times 2 \times 2 \times 2$
$24 = 4 \times 6$	$2 \times 2 \times 2 \times 3$

So the prime factors that were multiplied together to get 24 are $2 \times 2 \times 2 \times 3$.

Before we can actually begin to factor we must have an understanding of how to find the greatest common factor (or GCF).

For two or more integers the GCF will be the largest number which is a factor of all the integers in the problem.

EXAMPLE A: Find the GCF of 36 and 42

$36 = \boxed{2} \times 2 \times \boxed{3} \times 3$

$42 = \boxed{2} \times \boxed{3} \times 7$

6 is the largest integer which is a factor of both numbers.

The GCF is 6.

EXAMPLE B: Find the GCF of 18, 48, 54

$18 = \boxed{2} \times \boxed{3} \times 3$

$48 = \boxed{2} \times 2 \times 2 \times 2 \times \boxed{3}$

$54 = \boxed{2} \times \boxed{3} \times 3 \times 3$

2×3 or 6 is the largest number which is a factor of all three numbers.

The GCF is 6.

Another way to think of the GCF is that it is the largest number which will divide evenly into all of the numbers in the problem.

The GCF of the variable part of a problem is the largest power of each variable in all expressions being considered.

EXAMPLE C: Find the GCF of x^3y^2 and x^4y

$x^3y^2 = x \cdot x \cdot x \cdot y \cdot y$

$x^4y = x \cdot x \cdot x \cdot x \cdot y$

x^3 and y^1 are the highest power of x and y in both monomials.

The GCF is $x; y$

EXAMPLE D: Find the GCF of a^4bc^3 and a^2b^2

$a^4bc^3 = a \cdot a \cdot a \cdot a \cdot b \cdot c \cdot c \cdot c$

$a^2b^2 = a \cdot a \cdot b \cdot b$

a^2 and b are the highest powers of a and b in both monomials.

The GCF is a^2b

NOTE that the variable c is not part of the GCF because it is not common to both monomials.

EXAMPLE E: Find the GCF of $24x^4y^2$ and $18xy^3$

$24x^4y^2 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y$

$18xy^3 = 2 \cdot 3 \cdot 3 \cdot x \cdot y \cdot y \cdot y$

The GCF is $6xy^2$

NOTE the highest power of each variable that occurs in both monomials is always the lowest power of that variable. If you do not understand this, get help from your instructor or a tutor in the Math Center.

Given x^3 and x^2 the GCF is x^2 because the GCF must be able to divide into itself.

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Now let us use the GCF to factor a monomial from a polynomial.

EXAMPLE: Factor $3x^4 - 9x^2$

This means to find the monomial and the binomial that were multiplied to get the product $3x^4 - 9x^2$

1. First find the GCF.

   The largest number which will divide evenly into 3 and 9 is 3.
   The highest power of x that is in both terms is x^2.
   The GCF is $3x^2$.

2. Next divide each term in the binomial by the GCF.

   $$\frac{3x^4}{3x^2} - \frac{9x^2}{3x^2} = x^2 - 3$$

   The factored form is the product of the GCF ($3x^2$) and the quotient $(x^2 - 3)$.

   $$3x^4 - 9x^2 = 3x^2(x^2 - 3)$$

Factor $6a^2b^3 - 12ab^2 + 6b$

1. The GCF of the numerical coefficients (number parts) is 6.

2. The GCF of the variable parts is b.
   (**NOTE:** "a" is not common to all three terms.)

   $$\frac{6a^2b^3}{6b} - \frac{12ab^2}{6b} + \frac{6b}{6b} = a^2b^2 - ab + 1$$ Divide **each** term by the GCF.

   The factored form of $6a^2b^3 - 12ab^2 + 6b = 6b(a^2b^2 - 2ab + 1)$

Factor $8x^5y^3 - 4x^3y^4 - x^2y^5$

1. The GCF of the numerical coefficients is 1.
2. The GCF of the variable parts is x^2y^3

   $1 \cdot x^2 \cdot y^3 = x^2y^3$ so the GCF is x^2y^3

   $$\frac{8x^5y^3}{x^2y^3} - \frac{4x^3y^4}{x^2y^3} - \frac{x^2y^5}{x^2y^3} = 8x^3 - 4xy - y^2$$

   The factored form of $8x^5y^3 - 4x^3y^4 - x^2y^5$ is $x^2y^3(8x^3 - 4xy - y^2)$

EXERCISES:

a. $16x^4 - 8x^7$

b. $8y + 12$

c. $14x^2 - x$

d. $y^3 + 4y^2 + 3y$

e. $3a^2b^2 - 9ab^2 + 15b^2$

f. $3a^2 - 2b^3 + 7$

g. $14x^3y + 28x^2y - 3xy$

h. $20a^2b^3 - 15ab^3 + 5ab^2$

i. $x^4y^4 - 3x^3y^3 + 6x^2y^2$

j. $16x^2y - 8x^3y^4 - 48x^2y^2$

KEY:

a. $8x^4(2 - x^3)$

b. $4(2y + 3)$

c. $x(14x - 1)$

d. $y(y^2 + 4y + 3)$

e. $3b^2(a^2 - 3x + 5)$

f. no common factors (GCF = 1)

g. $xy(14x^2 + 28x - 3)$

h. $5ab^2(4ab - 3b + 1)$

i. $x^2y^2(x^2y^2 - 3xy + 6)$

j. $8x^2y(2 - xy^3 - 6y)$

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