Solving Radical Equations

Radical equations are equations contain radical expressions. The radical equations we are going to solve are mainly square root equations and cubic root equations.

Example #1: Solve $\sqrt{x} = 8$

Solution:

The first thing we need to do to solve radical equations is to remove the radical (*n*th roots).

$\sqrt{x} = 8$ — To remove the square root on the left side, we will need to square both sides of the equation.

$\left(\sqrt{x}\right)^2 = (8)^2$ — Simplify each side of the equation.

$x = 64$

$\sqrt{64} = 8 \quad 8 = 8$ ✓ — Check the answer. $x = 64$ is the solution.

Example #2: Solve $\sqrt{2x-5} = 3$

Solution:

This equation looks a little different than the previous one. The **radicand** (the expression under the radical sign) of the previous equation is *x*. The radicand of this equation is $2x-5$. But, as long as the **radical term** is isolate, we can follow the same steps to solve the equation as mentioned above.

$\sqrt{2x-5} = 3$ — To remove the square root on the left side, we will need to square both sides of the equation.

$\left(\sqrt{2x-5}\right)^2 = (3)^2$ — Simplify each side of the equation. Solve for *x*.

$2x - 5 = 9 \quad 2x = 14$

$x = 7$

$\sqrt{2(7)-5} = 3$ — Check the answer. $x = 7$ is the solution.

$\sqrt{9} = 3 \quad 3 = 3$ ✓

Example #3: Solve $\sqrt{2x+8} = x$

Solution:

$$\sqrt{2x+8} = x$$

To remove the square root on the left side, we will need to square both sides of the equation.

$$\left(\sqrt{2x+8}\right)^2 = (x)^2$$
$$2x+8 = x^2$$

Simplify each side of the equation.
Solve for *x*.

$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x = 4 \text{ or } x = -2$$

To solve a quadratic equation, we need to set one side of the equation equal to zero. Then factor the equation.

$$\sqrt{2(4)+8} = 4$$
$$\sqrt{16} = 4$$
$$4 = 4 \ ✓$$

We have to check the solutions to see if they work.

When substitute 4 into the equation, we receive a true statement. Therefore 4 is a solution.

$$\sqrt{2(-2)+8} = -2$$
$$\sqrt{4} = -2$$
$$2 \neq -2 \ ✗$$

When substitute -2 into the equation, the result is not a true statement. So -2 is not a solution.

$x = 4$ is the solution

Example #4: Solve $\sqrt{4-x}+5=8$

Solution:

The radical term in this equation $\sqrt{4-x}$ is not isolated (not by itself). So we have to isolate (remove +5) the radical term before we can follow the same steps to solve the equation as mentioned above.

$\sqrt{4-x}+5=8$

$\sqrt{4-x}+5-5=8-5$

To remove +5 on the left side, we will need to subtract 5 on both sides of the equation.

$\sqrt{4-x}=3$

$\left(\sqrt{4-x}\right)^2=(3)^2$

Now the radical is isolated. To remove the square root on the left side, we will need to square both sides of the equation.

$4-x=9$

$x=-5$

Simplify each side of the equation.
Solve for *x*.

$\sqrt{4-(-5)}=3$

$\sqrt{9}=3$

$3=3$ ✓

We have to check the solution to see if it works.

$x=-5$ is the solution.

Example #5: Solve $\sqrt{4-y}=y-2$

Solution:

$\left(\sqrt{4-y}\right)^2=(y-2)^2$

$4-y=(y-2)(y-2)$

The radical is isolated. We will need to square both sides of the equation to remove the square root on the left side.

$4-y=y^2-4y+4$

$y^2-3y=0$, $y(y-3)=0$

$y=0$ or $y=3$

We need to FOIL the right side and simplify the equation.
Solve for *y*.

$\sqrt{4-(0)}=0-2$

$\sqrt{4}=2\neq-2$ X

$\sqrt{4-(3)}=3-2$

$\sqrt{1}=1=1$ ✓

We have to check the solution to see if it works.

$y=3$ is the solution.

Example #6: Solve $\sqrt[3]{x} = -4$

Solution:

The first thing we need to do to solve this radical equation is to remove the radical (*n*th roots).

$$\left(\sqrt[3]{x}\right)^3 = (-4)^3$$
$$x = -64$$

The radical is isolated. We will need to cube both sides of the equation to remove the cubic root on the left side.

$$\sqrt[3]{-64} = -4 \checkmark$$

We have to check the solution to see if it works.

$x = -64$ is the solution.

Example #7: Solve $\sqrt[3]{x+10} = 4$

Solution:

$$\sqrt[3]{x+10} = 4$$
$$\left(\sqrt[3]{x+10}\right)^3 = (4)^3$$
$$x+10 = 64$$
$$x = 54$$

The radical is isolated. We will need to **cube** both sides of the equation to remove the cubic root on the left side.

$$\sqrt[3]{54+10} = \sqrt[3]{64} = 4 \checkmark$$

We have to check the solution to see if it works.

$x = 54$ is the solution.

Exercises: Solve the following radical equations

1. $\sqrt{3y} - 1 = 5$ **2.** $\sqrt[3]{x-4} = -2$ **3.** $x - 1 = \sqrt{5x-9}$ **4.** $\sqrt{d} + 6 = d$ **5.** $\sqrt{x-1} = x - 7$

Answers:

1. $\{12\}$ **2.** $\{-4\}$ **3.** $\{5,2\}$ **4.** $\{9\}$ **5.** $\{10\}$