Solving Equations in the Form x + a = b

To *solve* an equation means to find the value of the variable so that the original equation is true when the variable is replaced with the value.

EXAMPLE: $x + 3 = 8$

If x is replaced with 5, the equation is true.

$$x + 3 = 8$$
$$\downarrow$$
$$5 + 3 = 8$$
$$8 = 8 \text{ which is true}$$

To solve equations, we will use the following properties:

Addition Property of Equations.
The same number can be added to each side of an equation without changing the solution.

If a = b, then $a + c = b + c$ and the solution stays the same.

Addition Property of Opposites
The sum of a term and its opposite is zero.

$$5 + (-5) = 0$$
$$-4 + 4 = 0$$
$$\frac{2}{3} + \left(-\frac{2}{3}\right) = 0$$
$$a + (-a) = 0$$

Addition Property of Zero
The sum of a term and zero is the term

$$5 + 0 = 5$$
$$0 + (-4) = -4$$
$$a + 0 = a$$

In equations of the form $x + a = b$, x is a variable which represents an unknown number and a and b are constants.

EXAMPLES: $\underline{x + a = b}$

$$x + 3 = 8$$
$$x - 5 = -6$$

NOTE that $x - 5 = -6$ still fits the form $x + a = b$, though the operation is subtraction and not addition. Remember that subtraction can be rewritten as addition of the opposite.

$$\underline{x + a = b}$$
$$x - 5 = -6$$
$$\downarrow \quad \downarrow$$
$$x + (-5) = -6$$

Our final goal in solving an equation is to have a statement where the variable is equal to the constant. The solution to the equation is the constant.

SOLVE: $x + 12 = -4$

To get x by itself on one side of the equation we must remove 12 from the left side of the equation. To do this we will add the **opposite of** 12 to both sides of the equation.

$$x + 12 = -4$$
$$x + \underbrace{12 + (-12)}_{} = \underbrace{-4 + (-12)}_{}$$

Now we will combine like terms: $x + 0 = -16$

Zero added to any number is the number itself, so $x + 0 = -16$ is the same thing as $x = -16$.
To check we will replace x with (-16) in the original equation.

$$x + 12 = -4$$
$$\downarrow$$
$$\underbrace{(-16) + 12}_{} = -4$$
$$-4 = -4 \quad \text{TRUE}$$

Be sure you understand each step. Get help if you don't understand.

SOLVE: $x - 4 = -6$

	Since $x - 4$ is equivalent to $x + (-4)$, you do not change the other side. Try to do this first step mentally!
$x + (-4) = -6$	
$x + (-4) + 4 = -6 + 4$	Add the opposite of -4 to both sides.
$x + 0 = -2$	
$x = -2$	
	CHECK: $-2 - 4 = -6$

EXAMPLE:

$$x-\frac{3}{8}=\frac{1}{2}$$

$$x-\frac{3}{8}+\frac{3}{8}=\frac{1}{2}+\frac{3}{8}$$ Add the opposite of ($-\frac{3}{8}$) to both sides.

$$x-0=\frac{1}{2}+\frac{3}{8}$$ Recall that to add fractions you **MUST** have a common denominator! The LCD is 8, so

$$\frac{1}{2}+\frac{3}{8}=\frac{1}{2}\cdot\frac{4}{4}+\frac{3}{8}=\frac{4}{8}+\frac{3}{8}$$

$$x=\frac{4}{8}+\frac{3}{8}$$

$$x=\frac{7}{8}$$

CHECK:

$$x-\frac{3}{8}=\frac{1}{2}$$

$$\frac{7}{8}-\frac{3}{8}=\frac{1}{2}$$

reduce the fraction $\rightarrow$ $\frac{4}{8}=\frac{1}{2}$

$$\frac{1}{2}=\frac{1}{2}$$ TRUE

NOTE that your goal is still to get *x* by itself by adding the opposite of the constant term to both sides.

EXAMPLE:

$$-5=9+x$$

$$-5+(-9)=9+(-9)+x$$

$$-14=x$$

CHECK:

$$-5 = 9 + x$$

$$\downarrow$$

$$-5 = 9 + (-14)$$

$$-5 = -5 \quad \text{TRUE}$$

EXERCISES: Solve and check.

1. $x - 4 = 11$ 2. $m + 9 = 2$ 3. $x + 7 = 7$ 4. $2 = x + 7$

5. $9 + a = -3$ 6. $y + \frac{3}{4} = -\frac{1}{4}$ 7. $x + \frac{1}{6} = -\frac{1}{3}$ 8. $\frac{4}{9} + a = -\frac{2}{9}$

9. $13 = -6 + m$ 10. $4 = -10 + y$

KEY:

1. $x = 15$ 2. $m = -7$ 3. $x = 0$ 4. $x = -5$ 5. $a = -12$
6. $y = -1$ 7. $x = -\frac{1}{2}$ 8. $x = -\frac{2}{3}$ 9. $m = 19$ 10. $y = 14$

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