Solving Rational Equations

In the following examples, you will see two types of rational equations (1-5) & (6). Remember, when solving the rational equation, we state all restricted values first.

Example (1): Solve $\frac{6}{x+1} = 3x$

Solution:

Since the expression ($x+1$) is in the denominator, $x=-1$ should not be the solution.

$$\frac{6}{x+1} = 3x$$

Now we want to **clear the fraction** to make our equation easier to solve by multiplying the denominator $x+1$ on each side.

$$\frac{6}{\cancel{x+1}}(\cancel{x+1}) = 3x(x+1)$$

On the left side, the denominator cancels out. On the right side, we need to use the distributive property to **remove the parenthesis**.

$$6 = 3x^2 + 3x$$
$$3x^2 + 3x - 6 = 0$$

Solve the quadratic equation by setting one side equal to 0.

$$3(x^2 + x - 2) = 0$$

Factor out the GCF, 3, before factoring.

$$\frac{\cancel{3}(x^2 + x - 2)}{\cancel{3}} = \frac{0}{3}$$

Since 3 is a constant, it won't affect our equation when we divide it out on both sides.

$$(x^2 + x - 2) = 0$$
$$(x-1)(x+2) = 0$$
$$x = 1 \text{ or } x = -2$$

Factor the remaining quadratic equation.

Set each factor equal to 0 and solve for x.

Since neither of the solutions are -1, the solutions of this equation are $x=1$ or $x=-2$

Example (2): Solve $\frac{9}{2x} - 5 = 2$

Solution:

Since the expression $2x$ is in the denominator, $x = 0$ should not be the solution.

$\frac{9}{2x} - 5 = 2$

Before we can clear out the fraction, we need to isolate the rational term by adding 5 on each side.

$\frac{9}{2x} = 7$

Now we can **clear the fraction** by multiplying the denominator $2x$ on each side.

$\frac{9}{\cancel{2x}}(\cancel{2x}) = 7(2x)$

On the left side, the denominator cancels out. On the right side, we **remove the parenthesis** by multiply 7 and $2x$.

$9 = 14x$

Solve the linear equation by dividing 14 on both sides.

$\frac{9}{14} = x$

Since the solution is not 0, the solution of this equation is $x = \frac{9}{14}$

Example (3): Solve $\frac{2x}{x+2}+\frac{3x}{x-1}=7$

Solution:

This problem is a little different than the previous two problems. There are two rational terms (fractions) on the left side. The two rational terms have two different denominators, so there will be two restricted values for this problem. Since the expression $x+2$ and $x-1$ are in the denominator, $x=-2$ and $x=1$ should not be the solutions.

$$\frac{2x}{x+2}+\frac{3x}{x-1}=7$$

In order to **clear both fractions** to make our equation easier to solve, we need to multiply the Least Common Denominator (LCD) on <u>each term</u>. LCD is $(x+2)(x-1)$

$$\frac{2x}{\cancel{x+2}}\cancel{(x+2)}(x-1)+\frac{3x}{\cancel{x-1}}(x+2)\cancel{(x-1)}$$
$$=7(x+2)(x-1)$$

On the left side, the denominators cancel out. On the right side, there is no denominator to cancel, so we just multiply 7 and $(x+2)(x-1)$

$$2x(x-1)+3x(x+2)=7(x+2)(x-1)$$

We need to use the distributive property to **remove the parentheses**.

$$2x^2-2x+3x^2+6x=7(x^2+x-2)$$
$$5x^2+4x=7x^2+7x-14$$
$$0=2x^2+3x-14$$

Remove all parentheses and <u>combine like terms</u> on each side.
Since it is a quadratic equation, we will set one side equal to zero and solve.

$$2x^2+3x-14=0$$
$$(2x+7)(x-2)=0$$
$$x=-\frac{7}{2} \quad \text{or} \quad x=2$$

Factor the quadratic equation.
Set each factor equal to 0 and solve for x.
Since neither of the solutions are -2 and 1, the solutions of this equation are $x=-\frac{7}{2}$ or $x=2$

Example (4): Solve $\dfrac{3}{x-2}+\dfrac{5}{x+2}=\dfrac{12}{x^2-4}$

Solution:

This problem is very similar to Example (3). The difference is that on the right side it is also a rational term. Therefore when we look for restricted values and LCD, we need to take the consideration of all three denominators. The factors of x^2-4 are $(x+2)(x-2)$, so there are still two restricted values for this problem. Since the expression $x+2$ and $x-2$ are in the denominator, $x=-2$ and $x=2$ should not be the solutions.

$$\frac{3}{x-2}+\frac{5}{x+2}=\frac{12}{x^2-4}$$

$$\frac{3}{x-2}+\frac{5}{x+2}=\frac{12}{(x+2)(x-2)}$$

In order to **clear fractions on both sides**, we need to multiply the Least Common Denominator (LCD) on <u>each term</u>.
LCD is $(x+2)(x-2)$

$$\frac{3}{\cancel{x-2}}(x+2)\cancel{(x-2)}+\frac{5}{\cancel{x+2}}\cancel{(x+2)}(x-2)$$
$$=\frac{12}{\cancel{(x+2)(x-2)}}\cancel{(x+2)(x-2)}$$

On the left side, the denominators cancel out.
On the right side, the denominator cancels out as well.

$$3(x+2)+5(x-2)=12$$
$$3x+6+5x-10=12$$

We need to use the distributive property to **remove the parentheses**.

$$8x-4=12$$
$$8x=16$$
$$x=2$$

Combine like terms on the left side.
Solve for x.
Since the solution is 2, we can't use this solution.
There is **no solution** for this problem. $\emptyset$

Example (5): Solve $\frac{x+1}{2x}-\frac{x-1}{4x}=\frac{1}{3}$

Solution:

This is the last problem we will be looking at that has two fractions on one side. Since there are three fraction terms, we need to take the consideration of all three denominators when looking for restricted value(s) and LCD. The only value that is restricted for x is 0.

$$\frac{x+1}{2x}-\frac{x-1}{4x}=\frac{1}{3}$$

In order to **clear fractions on both sides**, we need to multiply the Least Common Denominator (LCD) on <u>each term</u>. LCD is $12x$

$$\left(\frac{x+1}{\cancel{2x}}\right)\left(\overset{\mathbf{6}}{\cancel{12x}}\right)-\left(\frac{x-1}{\cancel{4x}}\right)\left(\overset{\mathbf{3}}{\cancel{12x}}\right)=\frac{1}{\cancel{3}}\left(\overset{4x}{\cancel{12x}}\right)$$

Both sides of the denominator cancel out when multiplying $12x$.

$$6(x+1)-3(x-1)=4x$$
$$6x+6-3x+3=4x$$

We need to use the distributive property to remove the parentheses. **Be aware of the sign changes when distributing the negative sign.**

$$3x+9=4x$$
$$9=x$$

Combine like terms on the left side. Solve for x. Since the solution is not 0, the solution of the equation is $x=9$

Example (6): Solve $\frac{x}{x+2}=\frac{4}{x-3}$

Solution:

This is a different type of rational equation. There is only one rational term on each side. The processes of solving this equation are the same as all previous problems. However, there is also a short cut First, we still need to look for restricted value(s). Since the expression $x+2$ and $x-3$ are in the denominator, $x=-2$ and $x=3$ should not be the solutions.

Method 1.

$$\frac{x}{x+2}=\frac{4}{x-3}$$

To **clear fractions on both sides**, we need to multiply the Least Common Denominator (LCD) on each term. LCD is $(x+2)(x-3)$

$$\left(\frac{x}{\cancel{x+2}}\right)\cancel{(x+2)}(x-3)=\frac{4}{\cancel{x-3}}(x+2)\cancel{(x-3)}$$

Both sides of the denominator cancel out when multiplying $(x+2)(x-3)$.

$$x(x-3)=4(x+2)$$
$$x^2-3x=4x+8$$
$$x^2-7x-8=0$$

We need to use the distributive property to remove the parentheses. **Since it is a quadratic equation, we need to set one side of the equation equal to 0.**

$$(x-8)(x+1)=0$$
$x=8$ or $x=-1$

Factor the quadratic equation and solve for x. The solution of the equation are $x=8$ or $x=-1$

Method 2.

$$\frac{x}{x+2}=\frac{4}{x-3}$$

Another way to solve this equation is to cross multiply. This would only work with one fraction term on each side.

$$x(x-3)=4(x+2)$$
$$x^2-3x=4x+8$$
$$x^2-7x-8=0$$

We need to use the distributive property to remove the parentheses. **Since it is a quadratic equation, we need to set one side of the equation equal to 0.**

$$(x-8)(x+1)=0$$
$x=8$ or $x=-1$

Factor the quadratic equation and solve for x. The solution of the equation are $x=8$ or $x=-1$

Exercises:

1. $\dfrac{2}{x+4} - 2 = \dfrac{x}{x+4}$

2. $\dfrac{4}{x+3} = \dfrac{6}{x-3}$

3. $\dfrac{1}{x} - \dfrac{1}{x^2} = \dfrac{1}{4}$

4. $\dfrac{6}{x+2} + \dfrac{3x}{x+2} = \dfrac{3}{x+2}$

5. $\dfrac{x}{x-3} - \dfrac{3}{2} = \dfrac{3}{x-3}$

Answers:

1. $x = -2$ **2.** $x = -15$ **3.** $x = 2$ **4.** $x = -1$ **5.** $\emptyset$