Translating Sentences Into Equations

To translate a sentence into an equation we must be able to break the sentence into 3 separate parts. Those 3 parts are as follows:
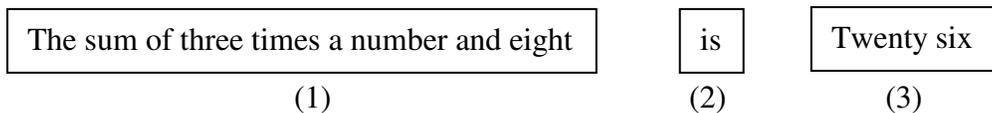
1. "a mathematical expression"
2. "an equal sign"
3. "an equivalent mathematical expression"

The following words and phrases are often used to mean "equals."

"is equal to"
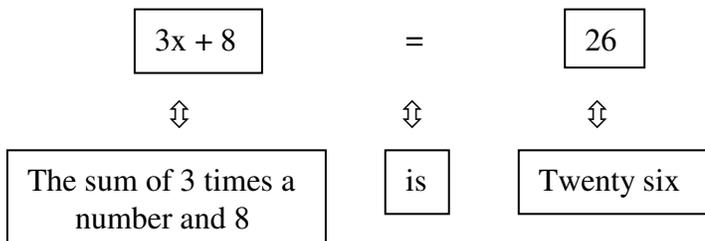"represents"
"is the same as"
"amounts to"
"was"
"is"

EXAMPLE: Translate and solve.

The sum of three times a number and eight is twenty-six. Find the number.

The sum of three times a number and eight		is		Twenty six
(1)		(2)		(3)

Now that we have broken the sentence into three parts we will translate. Find the equal sign first.

"is" translates to =

$3x + 8$	=	26
⇕	⇕	⇕
The sum of 3 times a number and 8	is	Twenty six

SOLVE: $3x + 8 = 26$

$$3x + 8 + (-8) = 26 + (-8)$$
$$3x = 18$$
$$\frac{1}{3} \cdot 3x = 18 \cdot \frac{1}{3}$$
$$x = 6$$

CHECK: $3(6) + 8 = 26$
              $18 + 8 = 26$ TRUE, if your translation is correct!
              $26 = 26$

EXAMPLE:　　　　Translate and solve.

Five times the difference between a number and four equals thirty. Find the number.

Five times the difference between a number and four equals thirty.

(1)		(2)	(3)
5 (x - 4)		=	30
five times the difference between a number and four		equals	[thirty]

SOLVE:
$$5(x - 4) = 30$$
$$5x - 20 = 30$$
$$5x - 20 + 20 = 30 + 20$$
$$5x = 50$$
$$\frac{1}{5} \cdot 5x = 50 \cdot \frac{1}{5}$$
$$x = 10$$

CHECK:
$$5[10 - 4] = 30$$
$$5[6] = 30$$
$$30 = 30 \qquad \text{TRUE}$$

Another check: ASK: "Is five times the difference between 10 and 4 equal to 30?"

EXAMPLE:　　　　Translate and solve.

The sum of two numbers is thirteen. The difference between four times the larger number and five equals the sum of five times the smaller number and two. Find the two numbers.

The first sentence is only giving you information about two two numbers. It is <u>not</u> part of your equation.

The equation is the next part:

The difference between four times the larger number and five EQUALS the sum of five times the smaller number and two.

The information that the sum of the two numbers is thirteen will be used to describe the two numbers.

　　　1st number　　+　　2nd number　　　　=　　　13
　　　　　x　　　　　　　　(13 - x)

If we call one of the numbers "x", the other number must be the total (13) minus x or (13 - x)

Suppose x was 3 then 3 + (13 - 3) = 13
$$3 + 10 = 13$$
$$13 = 13$$

Suppose x was 8 then 8 + (13 - 8) = 13
$$8 + 5 \quad = 13$$
$$13 \quad = 13$$

Suppose x was 4 then 4 + (13 - 4) = 13
$$4 + 9 \quad = 13$$
$$13 \quad = 13$$

The problem is we don't know what the number really is, so we call it x. Therefore, we don't know what number to subtract from 13, so we write (13 – x).

Let's call the smaller number x and the larger number (13 – x).

Our equation will be:

4 (larger number) - 5		5 (smaller number) + 2

=

The difference between four times the larger number and five.

The sum of five times the smaller number and two.

Smaller number = x
Larger number = (13 - x)

4(larger number) - 5 = 5(smaller number) + 2
$$4(13 - x) \quad - 5 = 5x + 2$$

$$4(13 - x) \ - \ 5 \ = \ 5x + 2$$
$$52 - 4x \ - 5 \ = \ 5x + 2$$
$$52 - 5 \ - 4x \ = \ 5x + 2$$
$$47 \ - \ 4x \ = \ 5x + 2$$

$$47 - 4x + (-5x) \ = \ -5x + 5x + 2$$
$$47 \ - 9x \ = \ 2$$
$$-47 + \ 47 \ - 9x \ = \ 2 + (-47)$$
$$-9x \ = \ -45$$

$$(-\frac{1}{9}) \, (-9x) \ = \ (-45) \, (-\frac{1}{9})$$
$$x = \ 5$$

Smaller number =   x   = 5
Larger number = 13 - 5 = 8

CHECK: 4(8) - 5 = 5(5) + 2
$$32 \quad - 5 = 25 \quad + 2$$
$$27 = \ 27 \qquad \text{TRUE}$$

This particular type of problem is difficult, but this concept occurs many times in algebra problems.

REMEMBER that the first piece of information that tells you what the sum is, is **not** part of your equation. You will use this to **describe** the two numbers.

The sum of two numbers is 15.
The numbers are x and 15 - x.

The sum of two numbers is 25.
The numbers are x and 25 - x.

The sum of two numbers is 54.
The numbers are x and 54 - x.

EXERCISES: Translate and solve.

1.   The sum of a number and twelve is ten. Find the number.

2.   The difference between twelve and the product of five and a number equals seven. Find the number.

3.   Five times the difference between a number and four equals thirty. Find the number.

4.   Two thirds of a number is six.

5.   The sum of two numbers is twenty-four. The difference between twice the larger number and eleven equals the difference between four times the smaller number and seventeen. Find the number.

6.   The sum of two numbers is thirty-five. The product of four and the smaller number is three times the larger number.

KEY:

1. $x + 12 = 10$; $x = -2$ 2. $12 - 5x = 7$; $x = 1$

3. $5(x - 4) = 30$; $x = 10$ 4. $\dfrac{2}{3}x = 6$; $x = 9$

5. $2(24 - x) - 11 = 4x - 17$; 15 & 9   6. $4x = 3(35 - x)$; 15 & 20

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