

## MAC 1105 Solving Quadratic Equations by Completing the Square

**Quadratic Equation** – Any equation which can be written in the form of  $ax^2 + bx + c = 0$ .

**Completing the Square** – Solving a Quadratic Equation by creating a Polynomial which can be factored as a Perfect Square Trinomial.

**Example:** Solve  $x^2 + 2x - 8 = 0$  by Completing the Square.

**a      b      c**

**Step 1:** Move c to the opposite side of the equation.

$$x^2 + 2x - 8 = 0 \rightarrow x^2 + 2x = 8$$

$$+ 8 \quad + 8$$

**Step 2:** Add  $\left(\frac{b}{2}\right)^2$  to each side of the equation. **b = 2**

$$\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1^2 = 1 \quad \rightarrow \quad x^2 + 2x + 1 = 8 + 1 \quad \rightarrow \quad x^2 + 2x + 1 = 9$$

**Step 3:** Factor  $x^2 + 2x + 1$  as a Perfect Square Trinomial. (Remember  $\sqrt{\quad}$  and squares are inverses.)

$$x^2 + 2x + 1 = 9$$

$$(x + 1)(x + 1) = 9$$

$$(x + 1)^2 = 9$$

**Step 4:** Square Root both sides of the equation.

$$\sqrt{(x + 1)^2} = \pm\sqrt{9} \quad \rightarrow \quad x + 1 = \pm 3$$

**Step 5:** Solve for x.

$$x + 1 = 3$$

$$- 1 \quad - 1$$

$$x = 2$$

$$x + 1 = -3$$

$$- 1 \quad - 1$$

$$x = -4$$

**Example:** Solve  $2x^2 - 5x - 3 = 0$  by Completing the Square.

**Step 1:** Use Algebra to move c to the opposite side of the equation.

$$2x^2 - 5x - 3 = 0 \rightarrow 2x^2 - 5x = 3$$
$$+ 3 + 3$$

**Step 2:** Divide both sides of the equation by a. (You want the leading coefficient to be 1.)

$$\frac{2}{2}x^2 - \frac{5}{2}x = \frac{3}{2}$$

**Step 3:** Add  $\left(\frac{b}{2}\right)^2$  to each side of the equation.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{\frac{5}{2}}{2}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16} \rightarrow x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16} \rightarrow x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{49}{16}$$

**Step 4:** Factor  $x^2 - \frac{5}{2}x + \frac{25}{16}$  as a Perfect Square Trinomial.

$$x^2 - \frac{5}{2}x + \frac{25}{16} = \left(x - \frac{5}{4}\right)^2 \rightarrow \left(x - \frac{5}{4}\right)^2 = \frac{49}{16}$$

**Step 5:** Square Root both sides of the equation.

$$\sqrt{\left(x - \frac{5}{4}\right)^2} = \pm \sqrt{\frac{49}{16}} \rightarrow x - \frac{5}{4} = \pm \frac{7}{4}$$

**Step 6:** Solve for x.

$$x - \frac{5}{4} = \frac{7}{4} \qquad x - \frac{5}{4} = \frac{-7}{4}$$
$$+ \frac{5}{4} + \frac{5}{4} \qquad + \frac{5}{4} + \frac{5}{4}$$
$$x = 3 \qquad x = -\frac{1}{2}$$

**Practice Problems:**

Solve the following Quadratic Equations by Completing the Square:

- 1)  $x^2 + 4x - 21 = 0$       **Solution:**  $x = 3, -7$
- 2)  $x^2 - 12x = -20$       **Solution:**  $x = 2, 10$
- 3)  $3x^2 - 5x + 2 = 0$       **Solution:**  $x = \frac{2}{3}, 1$
- 4)  $4x^2 + 19x = -12$       **Solution:**  $x = -\frac{3}{4}, -4$