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LEARNING **COMMONS**

Special Factoring

In this lab assignment we will look at two special kinds of factoring. One kind is the difference of two squares, which is the product of the sum and difference of two terms.

EXAMPLES: $a^2 - b^2 = (a + b)(a - b)$
$x^2 - 4 = (x + 2)(x - 2)$

The other kind is the factoring of perfect square trinomials which are the result of squaring a binomial.

EXAMPLES: $a^2 + 2ab + b^2 = (a + b)^2$

$x^2 + 14x + 49 = (x + 7)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

$x^2 - 12x + 36 = (x - 6)^2$

We will look at how we factor the difference of two squares first.

If the polynomial we are trying to factor has only two terms we need to check for two things:

1. Is the sign of the second term negative?

2. Are the terms of the polynomial <u>both</u> perfect squares? Another way to say this would be to ask if a number or variable was multiplied by itself to get that term.

EXAMPLE: $x^2 - 16$

The sign is negative indicating the <u>difference</u> of two squares.

x^2 and 16 are both perfect squares.

x^2 is a perfect square because $x \cdot x = x^2$

16 is a perfect square because $4 \cdot 4 = 16$

To factor $x^2 - 16$ we must first answer <u>yes</u> to both questions 1 and 2. We then take the square roots of x^2 and 16. The factorization of $x^2 - 16$ will be the <u>sum</u> and <u>difference</u> of the square roots.

$x^2 - 16$
square root is $x \rightarrow$ $(x)^2 - (4)^2$ $\leftarrow$ square root is 4
Factored form $\rightarrow$ $(x + 4)(x - 4)$

EXAMPLE: Factor: $y^2 - 25$

perfect square → $y^2 - 25$ ← perfect square

square root is → $(y)^2 - (5)^2$ ← square root is 5

Factored form → $(y + 5)(y - 5)$

EXAMPLE: FACTOR: $x^2 - 3$

perfect square → $x^2 - 3$ ← not a perfect square

*This is nonfactorable - We cannot factor this binomial.

EXAMPLE: Factor: $a^2 + 49$

*This is nonfactorable - we cannot factor this binomial, because the second term is positive. It is not a *difference* of two squares.

NOTE that we must answer yes to both questions (Is the sign negative? Are both terms perfect squares?) if the polynomial will factor into the sum and difference of two terms. If we say no to either question the polynomial is nonfactorable.

The second kind of special factoring is the factoring of perfect square trinomials. Perfect square trinomials are the result of squaring a binomial, or multiplying a binomial by itself.

$$x^2 + 10x + 25 = (x + 5)(x + 5) = (x + 5)^2$$
$$y^2 - 14y + 49 = (y - 7)(y - 7) = (y - 7)^2$$

Perfect square trinomials can be factored using trial and error methods of factoring, but it will save time if we can learn to recognize a perfect square trinomial and understand that it comes from squaring a binomial. If a trinomial is a perfect square trinomial then 3 things must be true:

1. The third term of the trinomial must be positive.

2. The first and third terms of the trinomial must be perfect squares.

3. The middle term must be twice the product of the square roots of the first and third term.

perfect square → $x^2 + 10x + 25$ ← perfect square

square root is x → $(x)^2 + 2(5)(x) + (5)^2$ ← square root is 5

the middle term is twice the product of x and 5

The factored form is the square of a binomial where the terms of the binomial are the square roots of the first and third term.

Factored form → $(x + 5)(x + 5)$ or $(x + 5)^2$

EXAMPLE: FACTOR: $4a^2 - 12ab + 9b^2$

perfect square → $4a^2 - 12ab + 9b^2$ ← perfect square
square root is $2a$ → $(2a)^2 - 2(2a)(3b) + (3b)^2$ ← square root is $3b$

Factored form → $(2a - 3b)(2a - 3b)$ or $(2a - 3b)^2$

FACTOR: $4y^2 - 36yz + 81z^2$

perfect square → $4y^2 - 36yz + 81z^2$ ← perfect square
square root is $2y$ → $(2y)^2 - 2(2y)(9z) + (9z)^2$ ← square root is $9z$

Factored form → $(2y - 9z)(2y - 9z)$ or $(2y - 9z)^2$

NOTE that all 3 things must be true if the trinomial is a perfect square trinomial. If we say no to any one of the 3, the trinomial is not a perfect square trinomial and we must use trial and error methods to factor.

FACTOR:

a. $a^2 - 4$

b. $x^2 - 6x + 9$

c. $y^2 - 81$

d. $4a^2 - 9b^2$

e. $9x^2 - 42x + 49$

f. $4y^2 - 20y + 25$

g. $x^2 + 16$

h. $9a^2 + 6a + 1$

i. $x^2 - 2$

j. $25y^2 + 10y + 1$

KEY:

a. $(a + 2)(a - 2)$

b. $(x - 3)^2$

c. $(y + 9)(y - 9)$

d. $(2a + 3b)(2a - 3b)$

e. $(3x - 7)^2$

f. $(2y - 5)^2$

g. nonfactorable

h. $(3a + 1)^2$

i. nonfactorable

j. $(5y + 1)^2$