

Simplifying Rational Expressions

A *rational expression* is a quotient of two monomials and/or polynomials.

Examples: $\frac{3x+1}{2}$, $\frac{x^2-1}{5x^2+2x}$, $\frac{3y}{5y^2}$, $\frac{1}{2}$, etc.

Beginning Steps:

1. Factor out the greatest common factor (GCF), if any, in all numerators and denominators.
2. Factor completely all numerators and denominators.
3. Remaining steps depend on the type of problem. See examples below for three types of problems: (I) single expression; (II) products and quotients; (III) sums and differences.

Type I: Single Rational Expression

Example #1 - Simplify: $\frac{w^2 + w - 12}{w^2 + 8w + 16}$.

Solution -

$$= \frac{w^2 + w - 12}{w^2 + 8w + 16}$$

$$= \frac{(w-3)(w+4)}{(w+4)(w+4)}$$

$$= \frac{(w-3)}{(w+4)}$$

1. there is no GCF in either numerator or denominator
2. factor numerator and denominator completely
3. cancel any like factors (representing 1)

Example #2 - Simplify: $\frac{3x + 6}{3x}$.

Solution -

$$= \frac{3x + 6}{3x}$$

$$= \frac{3(x + 2)}{3x}$$

$$= \frac{3}{3} \cdot \frac{(x + 2)}{x}$$

$$= \frac{x + 2}{x}$$

1. factor out GCF (of 3) in the numerator
2. there's no other factorization to be done
3. cancel like factors (representing 1)

Note- it is **incorrect** to cancel as follows: $\frac{3x + 6}{3x}$ 7 since the 3x in 3x+6 is not a factor but a term of the sum: 3x+6.

However, $\frac{3(x + 2)}{3x} = \frac{x + 2}{x}$ is correct.

Example #3 - Simplify: $\frac{150x - 6x^3}{6x^2 - 27x - 15}$.

Solution -

$$= \frac{150x - 6x^3}{6x^2 - 27x - 15}$$

$$= \frac{6x(25 - x^2)}{3(2x^2 - 9x - 5)}$$

$$= \frac{3 \cdot 2x(5 + x)(5 - x)}{3(2x + 1)(x - 5)}$$

$$= \frac{3 \cdot 2x(5 + x)(-1)(x - 5)}{3(2x + 1)(x - 5)}$$

$$= \frac{-2x(5 + x)}{2x + 1}$$

1. factor out GCF of 6x in numerator and of 3 denominator
2. factor numerator and denominator completely
3. cancel like factors (representing 1)

Note: $5 - x = -1(-5 + x) = -1(x - 5)$. (factoring out -1 and rewriting)

Type II: Products and Quotients

Example #4-

Simplify: $\frac{4x^2 - 9}{x + 1} \div \frac{10x^2 + 19x + 6}{x^2 + 8x + 7} \cdot \frac{5x + 10}{2x - 3}$

Solution -

$$\begin{aligned}
 &= \frac{4x^2 - 9}{x + 1} \div \frac{10x^2 + 19x + 6}{x^2 + 8x + 7} \cdot \frac{5x + 10}{2x - 3} \\
 &= \frac{4x^2 - 9}{x + 1} \cdot \frac{x^2 + 8x + 7}{10x^2 + 19x + 6} \cdot \frac{5x + 10}{2x - 3} && \text{(i) change division to mult. by the reciprocal} \\
 &= \frac{(2x + 3)(2x - 3)}{x + 1} \cdot \frac{(x + 7)(x + 1)}{(2x + 3)(5x + 2)} \cdot \frac{5(x + 2)}{2x - 3} && \text{(ii) factor completely and cancel} \\
 &= \boxed{\frac{5(x + 2)(x + 7)}{5x + 2}} && \text{(iii) write final answer as one fraction}
 \end{aligned}$$

Example #5 -

Simplify: $\frac{3x - 1}{2x^2 - 9x - 5} \cdot \frac{x^2 + 2x - 8}{6x - 2}$

Solution -

$$\begin{aligned}
 &= \frac{3x - 1}{2x^2 - 9x - 5} \cdot \frac{x^2 + 2x - 8}{6x - 2} \\
 &= \frac{3x - 1}{(2x + 1)(x - 5)} \cdot \frac{(x + 4)(x - 2)}{2(3x - 1)} && \text{(i) factor completely} \\
 &= \boxed{\frac{(x + 4)(x - 2)}{2(2x + 1)(x - 5)}} && \text{(ii) cancel like factors}
 \end{aligned}$$

Type III: Sums and Differences

Example #6 -

Add and simplify: $\frac{3y}{y^2 + 12y + 20} + \frac{y + 7}{y^2 - 100}$

Solution -

$$\begin{aligned}
 &= \frac{3y}{(y + 10)(y + 2)} + \frac{y + 7}{(y + 10)(y - 10)} && \text{(i) factor denominators \& find LCD} \\
 &= \frac{3y}{(y + 10)(y + 2)} \cdot \frac{(y - 10)}{(y - 10)} + \frac{y + 7}{(y + 10)(y - 10)} \cdot \frac{(y + 2)}{(y + 2)} && \text{(ii) LCD is } (y + 10)(y + 2)(y - 10) \\
 &= \frac{3y^2 - 30y}{(y + 10)(y + 2)(y - 10)} + \frac{y^2 + 9y + 14}{(y + 10)(y + 2)(y - 10)} && \text{(iii) write all fractions with LCD} \\
 &= \frac{3y^2 - 30y + y^2 + 9y + 14}{(y + 10)(y + 2)(y - 10)} && \text{(iv) add fractions} \\
 &= \boxed{\frac{4y^2 - 21y + 14}{(y + 10)(y + 2)(y - 10)}} && \text{(v) if possible, factor and reduce} \\
 & && \text{(vi) (cannot factor or reduce further)}
 \end{aligned}$$

Example #7 -

Subtract and simplify: $\frac{x + 3}{2x^2 + 13x + 6} - \frac{5}{6x + 3}$

Solution -

$$\begin{aligned}
 &= \frac{x + 3}{2x^2 + 13x + 6} - \frac{5}{6x + 3} && \text{(i) factor all denominators and find LCD} \\
 &= \frac{x + 3}{(2x + 1)(x + 6)} - \frac{5}{3(2x + 1)} && \text{(ii) LCD is: } 3(2x + 1)(x + 6) \\
 &= \frac{x + 3}{(2x + 1)(x + 6)} \cdot \frac{3}{3} - \frac{5}{3(2x + 1)} \cdot \frac{(x + 6)}{(x + 6)} && \text{(iii) write all fractions with LCD} \\
 &= \frac{3x + 9}{3(2x + 1)(x + 6)} - \frac{5x + 30}{3(2x + 1)(x + 6)} && \text{(iv) subtract fractions} \\
 &= \frac{3x + 9 - 5x - 30}{3(2x + 1)(x + 6)} = \boxed{\frac{-2x - 21}{3(2x + 1)(x + 6)}} && \text{(v) (cannot factor or reduce further)}
 \end{aligned}$$