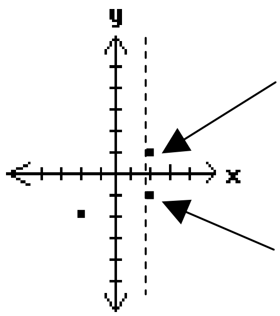
WILLIAM D. LAW, JR. LEARNING COMMONS

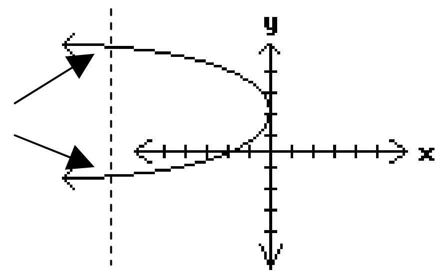
Functions, Domain and Range Overview

<u>The test for a function</u> (from a graph)

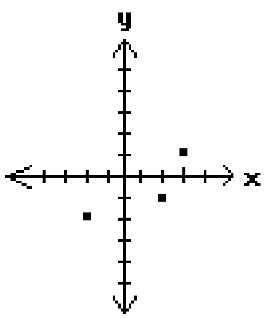
A *relation* is any set of ordered pairs (x,y). A *function* is a special type of relation. A ***function*** is a relation where each x-value has only one y-value. The ***vertical line test*** can be used to determine if the graph of a relation is a function. If a vertical line passes through more than one point <u>anywhere</u> on the graph, then it is <u>not</u> a function. See the examples below:



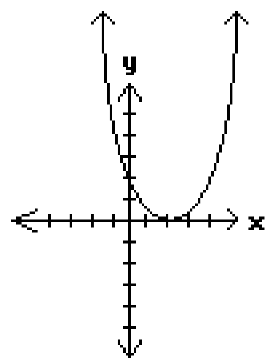
<u>NOT</u> a function: fails vertical line test



<u>NOT</u> a function: fails vertical line test



<u>IS</u> a function: passes vertical line test



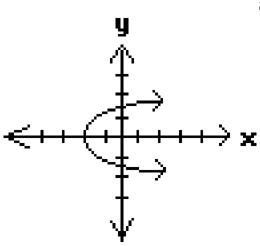
<u>IS</u> a function: passes vertical line test

The test for a function from its equation

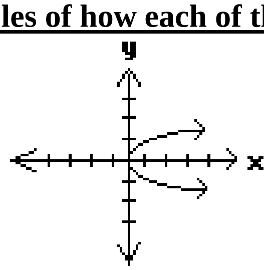
1. A *relation* is **NOT A FUNCTION** if there exists:

(i) <u>a "±" symbol on an x-expression</u> or
(ii) <u>even power of y</u> or
(iii) <u>y-variable expression inside absolute value symbols</u> or
(iv) <u>inequality symbols</u> (<, >, ≤, ≥)

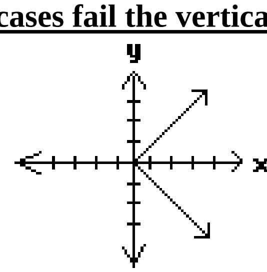
<u>Examples of how each of these 4 cases fail the vertical line test</u>



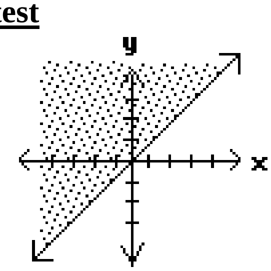
(i) $y = \pm\sqrt{x+2}$



(ii) $x=y^2$



(iii) $x=|y|$



(iv) $y \geq x$

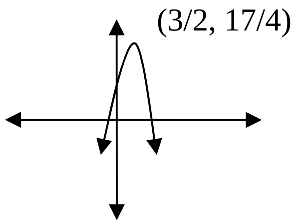
2. **<u>In ALL other cases the relation IS A FUNCTION.</u>**

This instructional aid was prepared by the Tallahassee Community College Learning Commons.

Determining the domain of a function from its equation

Domain deals with the acceptable values for the *x* variable and Range deals with the subsequent values for the *y* variable. Below are some examples that show some of the various types of problems most students encounter. Mainly two things limit your domain, a fraction and an even indexed radical. The range is probably easiest to determine when looking at a graph of the function.

1. $y = -x^2 + 3x + 2$

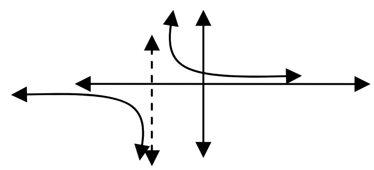


Hint: find highest or lowest point for range

Domain: no limits : $(-\infty, +\infty)$

Range: The y values have a peak but no bottom so the range is $(-\infty, 17/4]$

2. $y = \dfrac{1}{x+2}$



Hint: denominator can not equal 0; thus we set the bottom equal to 0; solve for *x*.

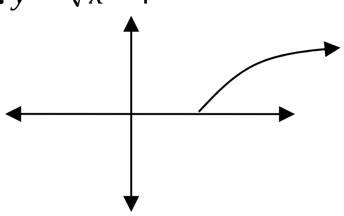
$x + 2 = 0$

$x = -2$

Domain: $(-\infty, -2) \cup (-2, +\infty)$

Range: $(-\infty, 0) \cup (0, +\infty)$

3. $y = \sqrt[4]{x-4}$



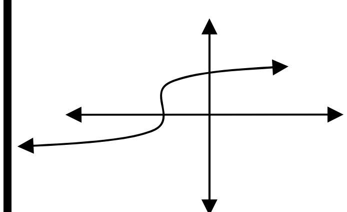
Hint: Even indexed roots have to be ≥ 0 if not in the denominator

Set the inside $x - 4 \geq 0$ and solve for *x*, thus $x \geq 4$

Domain: $[4, +\infty)$

Range: since y never gets less than 0, $[0, +\infty)$

4. $y = \sqrt[3]{x+3}$



Hint: no values of *x* will give undefined values, nor are any values of *y* not used.
* this will hold true for **any odd index.**

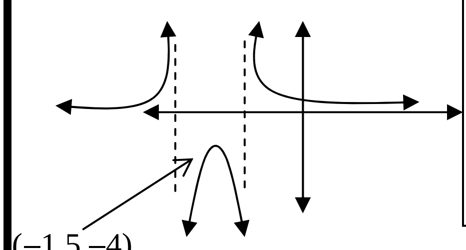
Domain: $(-\infty, +\infty)$

Range: $(-\infty, +\infty)$

5. $y = \dfrac{1}{x^2 + 3x + 2}$

Hint: set the bottom = 0 and solve by factoring.

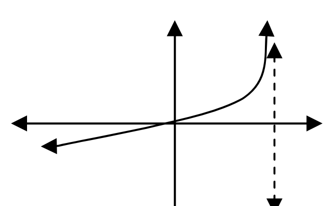
$x^2 + 3x + 2 = 0$,
$x + 1 = 0$,
$x = -1$ and
$x + 2 = 0$
$x = -2$
avoid $x = -1$ & -2



Domain: $(-\infty, -2) \cup (-2, -1) \cup (-1, +\infty)$

Range: $(-\infty, -4] \cup (0, +\infty)$

6. $y = \dfrac{3x}{\sqrt{4-x}}$



Hint: *compare to #3 & #2.

set inside of radical > 0 and solve for *x*; ***not = 0** since in the denominator.
$4 - x > 0$;
thus $4 > x$ or
$x < 4$

Domain: $(-\infty, 4)$ ($\textit{remember}: \quad x \neq 4$)

Range: $(-\infty, +\infty)$