WILLIAM D. LAW, JR.
LEARNING **COMMONS**

Composite Functions

Given two functions, combine them in a way such that the outputs of one function become the inputs for the other, making it a **composite function**.

$$(f \circ g)(x) = f\big(g(x)\big) \quad \textbf{OR} \quad (f \circ g)(x) = f \text{ "}composed\ of\text{" } g$$

Evaluating Composite Functions

Evaluate the function on the **right** side, and then substitute that result into the other function to find the answer.

Example: Given $f(x) = 5x - 3$ and $g(x) = x^2$, find $(f \circ g)(3)$.

Solutions:

Step 1: Set up the equation and start from the right side.

$(f \circ g)(3) = f\big(g(3)\big)$
Notice $g(3)$ is the input for $f(x)$,
so start by solving for g(3).

Given $g(x) = x^2$:
$g(3) = (3)^2$
$g(3) = 9$

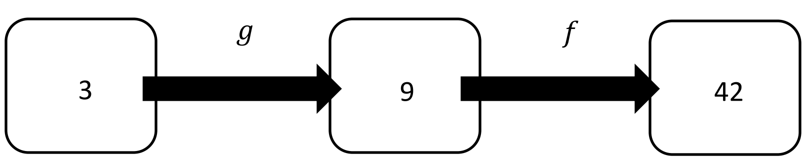
Step 2: Now substitute the answer for $g(3)$ into $f(x)$.

$f\big(g(3)\big) = f(9)$

Given $f(x) = 5x - 3$:
$f(9) = 5(9) - 3$
$f(9) = 42$

so $(\boldsymbol{f} \circ \boldsymbol{g})(\mathbf{3}) = \mathbf{42}$

$$g(3) \to f(9) \to 42$$



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Finding the Composite Function

To compose two functions, redefine the composition by using the definition to find $f(g(x))$ or $g(f(x))$.

Example: Given $f(x) = x^2 + 4$ and $g(x) = \frac{1}{x}$, find $(g \circ f)(x)$.

Solution:

Step 1: Set up the function using the definition.

$(g \circ f)(x) = g(f(x))$
Notice $f(x)$ is the input for $g(x)$, so start with $f(x)$.

Given $\boldsymbol{f(x) = x^2 + 4}$:
$g(\boldsymbol{f(x)}) = g(\boldsymbol{x^2 + 4})$

Step 2: Now substitute $x^2 + 4$ into $g(x)$ for every x. Simplify as needed.

Given $g(\boldsymbol{x}) = \frac{1}{\boldsymbol{x}}$:

$$g(\boldsymbol{x^2 + 4}) = \frac{1}{(\boldsymbol{x^2 + 4})}$$

so $\boldsymbol{g(f(x)) = \frac{1}{x^2 + 4}}$

Example: Given $f(x) = x^2 + 2x - 3$ and $g(x) = x + 1$ find $f(g(x))$.

Solution:

Since $f(g(x))$ uses $g(x)$ as the input for f, substitute $x + 1$ for $g(x)$ and simplify.

Step 1: Substitute.
$f(g(x)) = f(x + 1)$
$f(x + 1) = (x + 1)^2 + 2(x + 1) - 3$

Step 2: Simplify.
$f(x + 1) = (x^2 + 2x + 1) + 2x + 2 - 3$
$\boldsymbol{f(x + 1) = x^2 + 4x}$

Practice Exercises:

1. Given $f(x) = 2x - 6$ and $g(x) = x^2 + 3$, find $g(f(x))$.
2. Given $f(x) = 4 - x$ and $g(x) = x^3 - 1$, find $(f \circ g)(x)$.
3. Given $f(x) = 3x + 4$ and $g(x) = 2x$, find $(f \circ g)(5)$.
4. Given $f(x) = x + 7$ and $g(x) = \frac{1}{x^2 - 1}$ find $g(f(2))$.

Answers:

1. $g(f(x)) = 4x^2 - 24x + 39$
2. $f(g(x)) = 5 - x^3$
3. $f(g(5)) = 34$
4. $g(f(2)) = \frac{1}{80}$

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