

Multiplying Polynomials

Multiplying a Polynomial by a Monomial

To multiply a polynomial by a monomial we use a Distributive Property as well as the rule for multiplying exponential expressions.

EXAMPLE: $4x^2(x + 8)$

We will first multiply $4x^2$ and x . Then we will multiply $4x^2$ and 8 .

$$4x^2(x) + 4x^2(8)$$

$$4x^3 + 32x^2$$

REMEMBER to add the exponents if the bases are the same.

$$4x^2(x) = 4x^{2+1} = 4x^3$$

Also, REMEMBER that the sign we get when we multiply gives us the sign between the terms.

EXAMPLE: $-y(-3y^2 - 2y + 6)$

Use the Distributive Property to multiply each term inside the parentheses by $-y$. REMEMBER that the sign in front of the term goes with the term.

$$-y(-3y^2 - 2y + 6)$$

$$-y(-3y^2) - y(-2y) - y(6)$$

$$3y^3 + 2y^2 - 6y$$

REMEMBER that we cannot combine terms unless the variable parts are identical. This problem is simplified as far as possible.

EXAMPLE: $ab(2a^2 - 4ab - 6b^2)$

$$ab(2a^2) + ab(-4ab) + ab(-6b^2)$$

$$2a^3b - 4a^2b^2 - 6ab^3$$

Don't forget the rules for exponents!

Multiplying a Polynomial by a Polynomial

Multiplication of polynomials can be accomplished by using a horizontal format and the Distributive Property, or by using a vertical format. We will use the vertical format—the process is similar to multiplication of real numbers.

EXAMPLE: $(y^2 - 2y + 7)(y - 2)$

REWRITE in vertical format.

$$\begin{array}{r} y^2 - 2y + 7 \\ \underline{y - 2} \end{array}$$

Multiply each term by -2 :

$$\begin{array}{r} y^2 - 2y + 7 \\ \underline{y - 2} \\ -2y^2 + 4y - 14 \end{array}$$

Now multiply each term by y . Be sure to keep like terms lined up.

$$\begin{array}{r} y^2 - 2y + 7 \\ \underline{y - 2} \\ -2y^2 + 4y - 14 \\ \underline{y^3 - 2y^2 + 7y} \\ y^3 - 4y^2 + 11y - 14 \end{array} \quad \text{Now combine like terms}$$

If you compare multiplication of polynomials to long multiplication of integers you will see that the steps are very similar.

$$\begin{array}{r} 379 \\ \times 23 \\ \hline 1137 \end{array}$$

Multiply by 3

$$\begin{array}{r} 3x^2 + 3x - 5 \\ \underline{4x - 2} \\ -6x^2 - 6x + 10 \end{array}$$

Multiply by -2

$$\begin{array}{r} 379 \\ \times 23 \\ \hline 1137 \\ 758 \end{array}$$

Multiply by 2, keeping place values aligned

$$\begin{array}{r} 3x^2 + 3x - 5 \\ \underline{4x - 2} \\ -6x^2 - 6x + 10 \\ 12x^3 + 12x^2 - 20x \end{array}$$

Multiply by $4x$, keeping like terms together

$$\begin{array}{r} 379 \\ \times 23 \\ \hline 1137 \\ 758 \\ \hline 8717 \end{array}$$

Now add to get the total.

$$\begin{array}{r} 3x^2 + 3x - 5 \\ \underline{4x - 2} \\ -6x^2 - 6x + 10 \\ \underline{12x^3 + 12x^2 - 20x} \\ 12x^3 + 6x^2 - 26x + 10 \end{array}$$

Add by combining like terms

Sometimes there is a missing term in one of the polynomials. This means we must either leave a space or put in a place holder. We will use a place holder.

EXAMPLE: $(3a^3 - 5a^2 + 7)(6a - 1)$

Notice that the first polynomial has no “a” term. We’ll hold that place with “0a.”

$$\begin{array}{r}
 3a^3 - 5a^2 + 0a + 7 \\
 \underline{ 6a - 1} \\
 -3a^3 + 5a^2 - 0a - 7 \quad \leftarrow \text{multiply by } -1 \\
 \underline{18a^4 - 30a^3 + 0a^2 + 42a} \quad \leftarrow \text{multiply by } 6a \\
 18a^4 - 33a^3 + 5a^2 + 42a - 7 \quad \leftarrow \text{add}
 \end{array}$$

EXAMPLE: $(5x^3 - 5x + 2)(x - 4)$

There is no “x²” term in the first polynomial.

$$\begin{array}{r}
 5x^3 + 0x^2 - 5x + 2 \\
 \underline{ x - 4} \\
 -20x^3 + 0x^2 + 20x - 8 \quad \leftarrow \text{multiply by } -4 \\
 \underline{5x^4 + 0x^3 - 5x^2 + 2x} \quad \leftarrow \text{multiply by } x \\
 5x^4 - 20x^3 - 5x^2 + 22x - 8 \quad \leftarrow \text{add}
 \end{array}$$

EXERCISES: Multiply each of the following.

a. $-6a^3(a + 2)$

f. $(-x^2 + 3x - 2)(2x - 1)$

b. $2y(3y^2 - 4)$

g. $(3y^2 + 2y - 2)(5y - 4)$

c. $-x(-2x^4 - 3x^2 + 2)$

h. $(-2x^2 + 3x + 8)(x + 7)$

d. $(3y^3 - 2y^2 + 5)4y$

i. $(4a^3 - 2a + 5)(a + 6)$

e. $-3ab(a^2 - 5ab + 3b^2)$

j. $(5y^3 + 2y^2 - 9)(2y - 3)$

KEY:

a. $-6a^4 - 12a^3$

b. $6y^3 - 8y$

c. $2x^5 + 3x^3 - 2x$

d. $12y^4 - 8y^3 + 20y$

e. $-3a^3b + 15a^2b^2 - 9ab^3$

f. $-2x^3 + 7x^2 - 7x + 2$

g. $15y^3 - 2y^2 - 18y + 8$

h. $-2x^3 - 11x^2 + 29x + 56$

i. $4a^4 + 24a^3 - 2a^2 - 7a + 30$

j. $10y^4 - 11y^3 - 6y^2 - 18y + 27$