Find the equation of a logarithm function
given two points and the value of its vertical asymptote

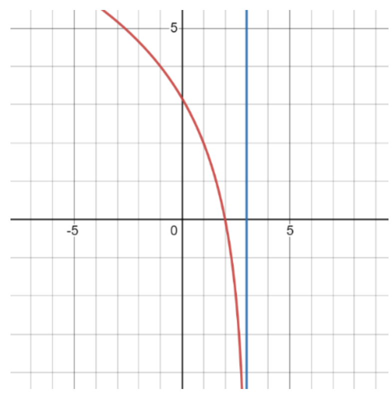
$$y = A\,log(x + B) + C$$

Important: One point should make the function argument (x + b) equal to 1.

Steps:

1. Look for the value of the vertical asymptote on the graph. This gives us the vertical asymptote, set $X - B = 0$. This gives the B value.
2. Look for reflection to the y axis. If the graph shows reflection, multiply the function argument $(x + b)$ by (-1).
3. Find C by using the point that makes the argument$(x + B)$ equal to 1. Therefore, $Alog(x + B)$ will be zero and C will be the value of y.
4. Substitute the coordinates of a second point from the graph into the function equation to find A. It will be $A = (y - C)/log(x + B)$
5. Rewrite the function equation in replacing A, B, and C with the values that were found.

Example1: Find the equation of the function for the graph below passing through (2,0), (1,2).



Solution: The general equation is $y = A\,log(x + B) + C$

1. The graph shows a vertical asymptote at x = 3. Therefore, B is given by $x + 3 = 0$. $B = -3$ the new equation is $y = A\,log(x - 3) + C$

2. The graph displays a reflection to the y axis. The function is $y = A\,log(-x + 3)$. The argument $(x - 3)$ was multiplied by (-1).

3. Now substitute (2,0) (choose the x value that makes the log argument equal to 1 for simplification) into the equation:

$$0 = \mathrm{A}\log(-2 + 3) + C,$$
$$0 = \mathrm{A}\log(1) + C.$$
$$therefore, C = 0.$$

The equation becomes: $y = A\log(-x + 3)$.

4. Substituting in the point (1,2):

$$2 = A\,log\,(-1 + 3),$$
$$2 = A\,log\,(2).$$
$$\text{Then } A = 2/log\,(2).$$

5. The final equation of the function is: $y = 2/log\,(2) * log(-x + 3).$

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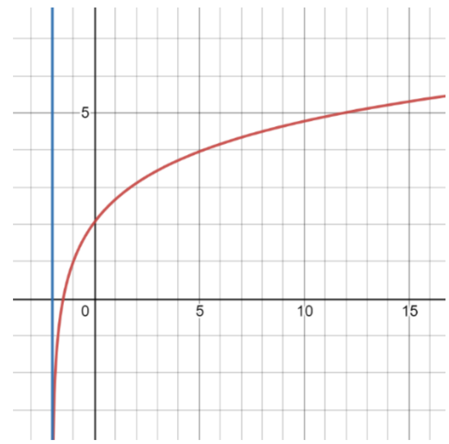
Example 2: Using the following graph, find the equation of the logarithm function passing through (-1,1) and (0, 2).



Solution: The general equation is

$$\boldsymbol{y = A\,log(x + B) + C}$$

1. The graph displays a Vertical Asymptote at $x = -2$. Thus setting $x - (-2) = 0$, gives us a B value of 2. The new equation is $y = A\,log(x + 2) + C$

2. No reflection was observed on the graph. Therefore, no change in the equation inside the parenthesis.

3. Now substitute (-1,1) (choose the x value that makes the log argument equal to 1 for simplification) into the equation that we have so far.

$$1 = A\,log\,(-1 + 2) + C,$$

$$1 = A\,log\,(1) + C; \;\; since\; log\,(1) = 0,$$

$$Therefore,\, C = 1.$$

The new equation is $y = A\,log(x + 2) + 1$

4. By substituting (0, 2) in the equation, we get:

$$2 = A\,log\,(0 + 2) + 1$$

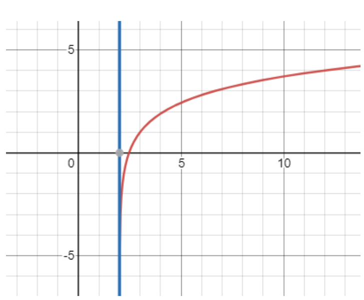
$$1 = A\,log\,(2)$$

$$A = (2 - 1)/log\,(2),$$

$$A = 1/log\,(2).$$

5. The final equation is $y = 1/log\,(2) * log(x + 2) + 1$

You try:



Solutions y = 3log(x – 2) + 1 ; (3,1) & (12,4) y = 2log(-x + 1) + 1; (-9,3) & (0,1)