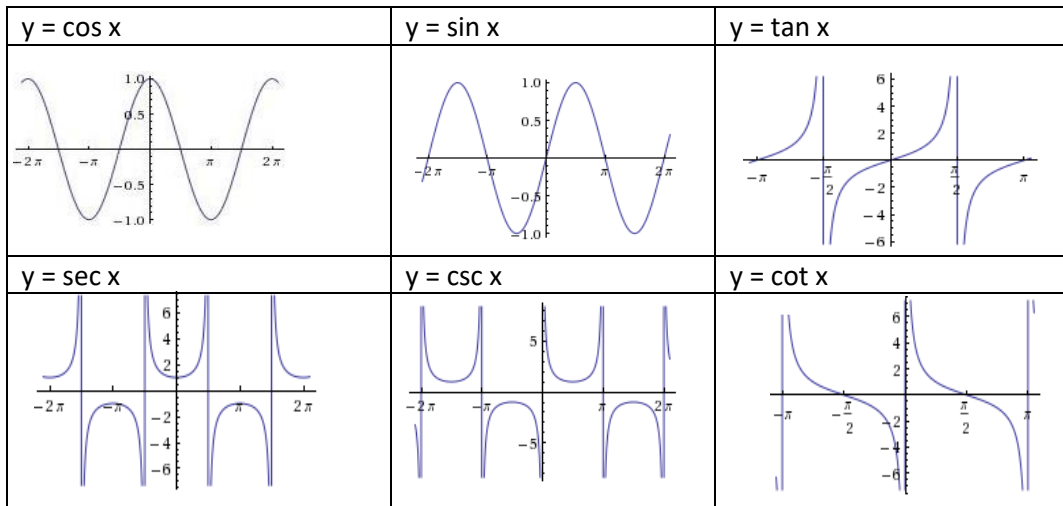


Basic Trigonometric Graphs:



Standard Forms

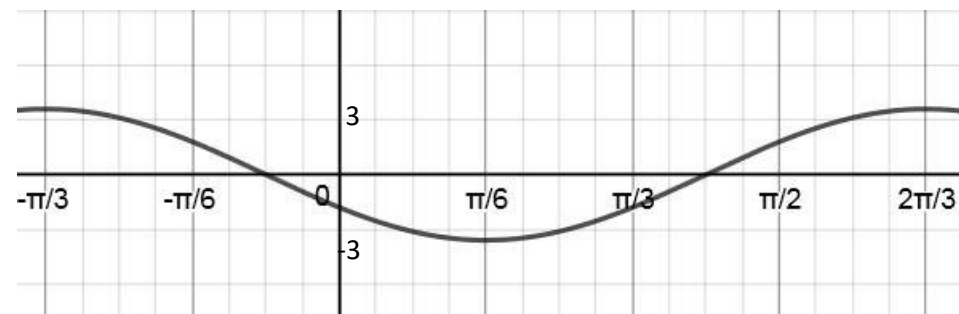
$y = a \sin k(x - b) + c$ $y = a \cos k(x - b) + c$	Amplitude= a	Period = $\frac{2\pi}{k}$, $k > 0$	Phase shift: b	Vertical shift: c
$y = a \csc k(x - b) + c$ $y = a \sec k(x - b) + c$	Not applicable	Period = $\frac{2\pi}{k}$, $k > 0$	Phase shift: b	Vertical shift: c
$y = a \tan k(x - b) + c$ $y = a \cot k(x - b) + c$	Not applicable	Period = $\frac{\pi}{k}$, $k > 0$	Phase shift: b	Vertical shift: c

Examples (these show one period for each example)

- $y = 3 \cos(2x + \frac{2\pi}{3})$: put it into the standard form by factoring out the 2 that is with the x. This gives:
 $y = 3 \cos 2(x + \frac{\pi}{3})$: Amplitude $\rightarrow 3$,
 Period $\rightarrow \frac{2\pi}{2} = \pi$ so would divide graph into $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and π .
 Phase shift $\rightarrow \frac{-\pi}{3}$ (to the left)

5 point method: (take the unshifted graph and adjust to get new points)

Starting point: unshifted $(0, 3) \rightarrow (0 - \frac{\pi}{3}, 3)$ Second point: unshifted $(\frac{\pi}{4}, 0) \rightarrow (\frac{\pi}{4} - \frac{\pi}{3}, 0)$
 Third point: unshifted $(\frac{\pi}{2}, -3) \rightarrow (\frac{\pi}{2} - \frac{\pi}{3}, -3)$ Fourth point: unshifted $(\frac{3\pi}{4}, 0) \rightarrow (\frac{3\pi}{4} - \frac{\pi}{3}, 0)$
 End of period: unshifted $(\pi, 3) \rightarrow (\pi - \frac{\pi}{3}, 3)$



angle	value
$-\frac{\pi}{3}$	3
$-\frac{\pi}{12}$	0
$\frac{\pi}{6}$	-3
$\frac{5\pi}{12}$	0
$\frac{2\pi}{3}$	3

2. $y = 2 \csc(2x + \frac{\pi}{2})$

put it into the standard form by factoring out the 2 that is with the x. This gives:

$y = 2 \csc 2(x + \frac{\pi}{4})$; period = $\frac{2\pi}{2} = \pi$ so would divide graph into $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and π .

Phase shift $\rightarrow -\frac{\pi}{4}$ (to the left)

5 point method: (take the unshifted graph and adjust to get new points)

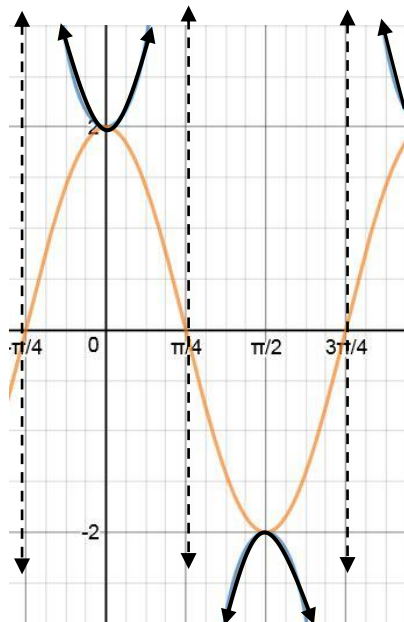
Starting point: unshifted $(0,1) \rightarrow (0 - \frac{\pi}{4}, 1)$

Second point: unshifted $(\frac{\pi}{4}, 0) \rightarrow (\frac{\pi}{4} - \frac{\pi}{4}, 0)$

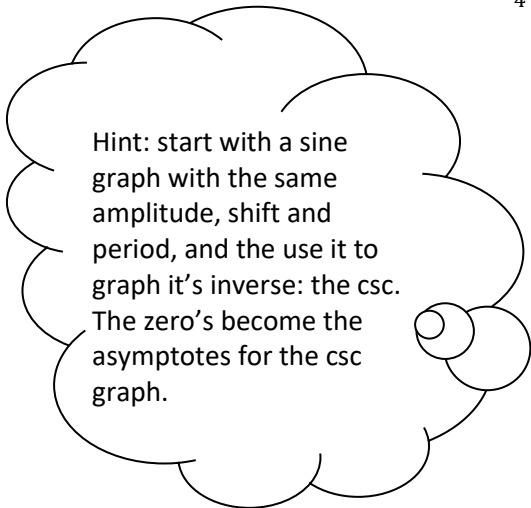
Third point: unshifted $(\frac{\pi}{2}, -1) \rightarrow (\frac{\pi}{2} - \frac{\pi}{4}, -1)$

Fourth point: unshifted $(\frac{3\pi}{4}, 0) \rightarrow (\frac{3\pi}{4} - \frac{\pi}{4}, 0)$

End of period: unshifted $(\pi, 1) \rightarrow (\pi - \frac{\pi}{4}, 1)$



angle	value
$-\frac{\pi}{4}$	asymptote
0	2
$\frac{\pi}{4}$	asymptote
$\frac{\pi}{2}$	-2
$\frac{3\pi}{4}$	asymptote



3. $y = \tan(x - \frac{\pi}{4})$

Period: no change since $k = 1 \rightarrow \pi$ so divide the graph into increments of $\frac{\pi}{4}$ like normal. Phase shift $\rightarrow \frac{\pi}{4}$

5 point method: (take the unshifted graph and adjust to get new points)

Starting point: unshifted $(-\frac{\pi}{2}, -\infty) \rightarrow (-\frac{\pi}{2} + \frac{\pi}{4}, -\infty)$ Second point: unshifted $(-\frac{\pi}{4}, -1) \rightarrow (-\frac{\pi}{4} + \frac{\pi}{4}, -1)$

Third point: unshifted $(0, 0) \rightarrow (0 + \frac{\pi}{4}, 0)$

Fourth point: unshifted $(\frac{\pi}{4}, 1) \rightarrow (\frac{\pi}{4} + \frac{\pi}{4}, 1)$

End of period: unshifted $(\frac{\pi}{2}, +\infty) \rightarrow (\frac{\pi}{2} + \frac{\pi}{4}, +\infty)$

angle	value
$-\frac{\pi}{4}$	$-\infty$
0	-1
$\frac{\pi}{4}$	0
$\frac{\pi}{2}$	1
$\frac{3\pi}{4}$	$+\infty$

