

## Using the Quotient Rule to find the Derivative

### The Process for the Quotient Rule:

1. Given  $H(x) = \frac{f(x)}{g(x)}$  then
2. Identify  $f(x)$ ,  $f'(x)$ ,  $g(x)$ , and  $g'(x)$
3.  $H'(x) = \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ ; Quotient Rule
4. Substitute and simplify

### Some other helpful rules:

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}[cf(x)] = c f'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1} dx$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x} dx$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(\sin x) = \cos x dx$$

$$\frac{d}{dx}(\cos x) = -\sin x dx$$

### Algebraic Example of Quotient Rule:

$$\text{Given: } H(x) = (x^2 + 1)/(x^3 + 6)^7$$

$$f(x) = x^2 + 1$$

$$f'(x) = 2x$$

$$g(x) = (x^3 + 6)^7$$

$$g'(x) = 7(x^3 + 6)^6(3x^2) = 21x^2(x^3 + 6)^6; \text{ using the chain rule}$$

$$H'(x) = \frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}; \text{ substitute the individual pieces}$$

$$H'(x) = \frac{(x^3+6)^7(2x) - (x^2+1)21x^2(x^3+6)^6}{[(x^3+6)^7]^2}; \text{ then simplify}$$

$$H'(x) = \frac{x(x^3+6)^6[(2)(x^3+6) - (x^2+1)21x]}{[(x^3+6)^7]^2}; \text{ factor out like terms before combining}$$

$$H'(x) = \frac{x(x^3+6)^6[(2)(x^3+6) - (x^2+1)21x]}{[(x^3+6)^7]^2};$$

$$H'(x) = \frac{x(x^3+6)^6[2x^3+12-21x^3-21x]}{(x^3+6)^{14}} \rightarrow \frac{x[12-19x^3-21x]}{(x^3+6)^8}$$

### Trig components Example:

$$\text{Given: } H(x) = \frac{\ln(x)}{x^2}$$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$g(x) = x^2$$

$$g'(x) = 2x$$

$$H'(x) = \frac{x^2(\frac{1}{x}) - \ln(x)(2x)}{[x]^2}; \text{ simplify}$$

$$H'(x) = \frac{x - \ln(x)(2x)}{[x^2]^2} \rightarrow \frac{x(1-2\ln(x))}{[x]^4} \rightarrow \frac{1-2\ln(x)}{[x]^3}$$

### Quotient Rule/Radical Example:

Given:  $H(x) = \frac{x^3+3}{\sqrt{x^2+7}}$ ; When you are working with radicals, the following exponent rules may come in handy:  $\sqrt[b]{m^a} = m^{\frac{a}{b}}$  and  $m^{-a} = \frac{1}{m^a}$

$$f(x) = x^3 + 3$$

$$f'(x) = 3x^2$$

$$g(x) = (x^2 + 7)^{\frac{1}{2}}$$

$$g'(x) = \frac{1}{2}(x^2 + 7)^{-\frac{1}{2}}(2x) = x(x^2 + 7)^{-\frac{1}{2}} =$$

$$H'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}; \text{ so, we have:}$$

$$H'(x) = \frac{(x^2+7)^{\frac{1}{2}}(3x^2) - (x^3+3)(x(x^2+7)^{-\frac{1}{2}})}{[(x^2+7)^{\frac{1}{2}}]^2}; \text{ then we begin simplifying and re-writing (if necessary)}$$

$$H'(x) = \frac{\sqrt{(x^2+7)}(3x^2) - (x^3+3)\left[\frac{x}{\sqrt{(x^2+7)}}\right]}{x^2+7}; \text{ getting a common denominator}$$

$$H'(x) = \frac{\sqrt{(x^2+7)}(3x^2)\left[\frac{\sqrt{(x^2+7)}}{\sqrt{(x^2+7)}}\right] - (x^3+3)\left[\frac{x}{\sqrt{(x^2+7)}}\right]}{x^2+7}; \text{ continuing to simplify}$$

$$H'(x) = \frac{\left[\frac{(3x^2)(x^2+7)}{\sqrt{(x^2+7)}}\right] + \left[\frac{-x^4-3x}{\sqrt{(x^2+7)}}\right]}{x^2+7}; \text{ continuing to simplify}$$

$$H'(x) = \frac{\left[\frac{2x^4+21x^2-3x}{\sqrt{(x^2+7)}}\right]}{x^2+7} \rightarrow \left[\frac{2x^4+21x^2-3x}{\sqrt{(x^2+7)}}\right] \cdot \left[\frac{1}{x^2+7}\right] \rightarrow \frac{2x^4+21x^2-3x}{(x^2+7)^{\frac{3}{2}}}$$

### You try's:

1.  $H(x) = \frac{1}{\sqrt{25-y^2}}$
2.  $H(x) = \frac{\sec^2 x}{1+x^2}$
3.  $H(x) = \frac{x-3}{x+7}$
4.  $H(t) = \frac{\ln x}{(x+1)^4}$

### Solutions:

1.  $H'(x) = \frac{y}{(25-y^2)^{\frac{3}{2}}}$
2.  $H'(x) = \frac{2\sec^2(x)[\tan(x)-x+\tan(x)]}{(1+x^2)^4}$
3.  $H'(x) = \frac{10}{(x+7)^2}$
4.  $H'(x) = \frac{\frac{x+1}{x} - 4 \ln(x)}{(x+1)^5}$