Finding the Key Components of Circles

Given an equation of a circle, identify the key components: center and radius

Circle formula:

\[ r(\text{radius}), \text{center}(h,k) \]

\[ (x - h)^2 + (y - k)^2 = r^2 \]

Example 1. Given an equation of a circle with no visible (h,k) values.

\[ x^2 + y^2 = 25 \]

Knowing the basic formula of a circle allows us to compare each area and solve for what we need to know. We can start by solving for the radius. We can see that:

\[ r^2 = 25. \]

\[ \sqrt{r^2} = \sqrt{25} \]

\[ r = 5. \]

Now we need to find the center point (h,k). We compare the equation we were given to the basic circle formula to find this. We have:

\[ x^2 \leftrightarrow (x - 0)^2, \]

Thus, our h value must be 0.

Let’s do the same with the y component.

\[ y^2 \leftrightarrow (y - 0)^2, \]

Thus, our k value must be 0 also.

Now we know that our center (h,k) is at (0,0) and our radius is 5.

Example 2. Given an equation where both groups have a negative sign.

\[ (x - 1)^2 + (y - 2)^2 = 17 \]

So, let’s follow the same process as in Example 5. We can start by finding our radius.

\[ r^2 = 17. \]

\[ \sqrt{r^2} = \sqrt{17} \]

\[ r = \sqrt{17} \]

Next, we can compare each of our groups to the patterns set by the basic circle formula. We have:

\[ (x - 1)^2 \leftrightarrow (x - h)^2, \]

we can see that h = 1.

\[ (y - 2)^2 \leftrightarrow (y - k)^2, \]

we can see that k = 2.

Now we know that our center (h,k) is at (1,2) and our radius is \( \sqrt{17} \).
Example 3. Given an equation where groups may have a negative or positive sign.

\[(x - 2)^2 + (y + 3)^2 = 4\]

We are going to follow the same step by step process.

\[r^2 = 4,\]
\[\sqrt{r^2} = \sqrt{4},\]
\[r = 2\]

Next, we can compare each of our groups to the patterns set by the basic circle formula. We have:
\[(x - 2)^2 \iff (x - h)^2,\]
we can see that \(h = 2\).

\[(y + 3)^2 \iff (y - (-3))^2 \iff (y - k)^2,\]
we can see that \(k = -3\).

Now we know that our center \((h,k)\) is at \((2, -3)\) and our radius is 2.

You Try:

Examples:
1. \(x^2 + (y - 2)^2 = 25\)
2. \((x - 2)^2 + (y - 3)^2 = 17\)
3. \((x + 2)^2 + (y + 3)^2 = 9\)

Answers:
1. Center at \((0, 2)\) and radius = 5
2. Center at \((2,3)\) and radius = \(\sqrt{17}\)
3. Center at \((-2, -3)\) and radius is 3.