WILLIAM D. LAW, JR. LEARNING COMMONS

Graphing Linear Equations

A linear equation has infinitely many ordered pair solutions. The graph of an equation in two variables is a drawing of the ordered pair solutions of the equation. It is not possible to name *all* the solutions. We generally find three ordered pair solutions and graph them. The complete solution set can be shown by drawing a straight line through the graphs of the ordered pairs. An arrow on each end of the line shows that the solution set continues in both directions.

EXAMPLE: Graph $y = 3x + 2$

To find three ordered pair solutions, pick **any** three values for x and solve for y.

Let $x = 1$:

$y = 3(1) + 2$
$y = 3 + 2$
$y = 5$
$(1, 5)$

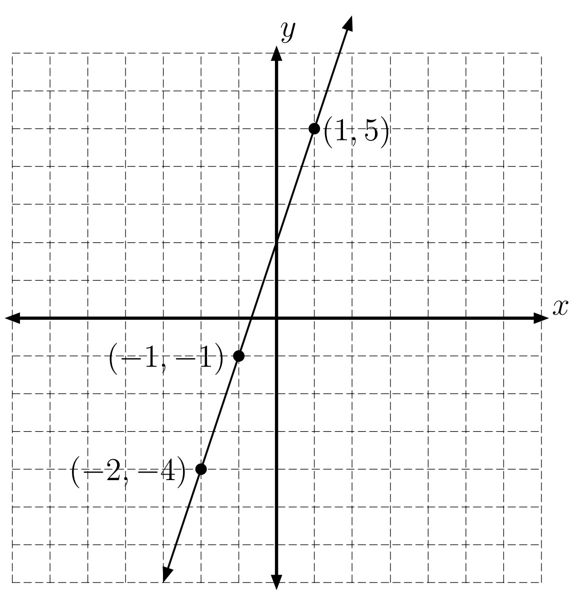
Let $x = -1$:

$y = 3(-1) + 2$
$y = -3 + 2$
$y = -1$
$(-1, -1)$

Let $x = -2$:

$y = 3(-2) + 2$
$y = -6 + 2$
$y = -4$
$(-2, -4)$

Now we graph the ordered pair solutions $(1, 5)$, $(-1, -1)$, and $(-2, -4)$.

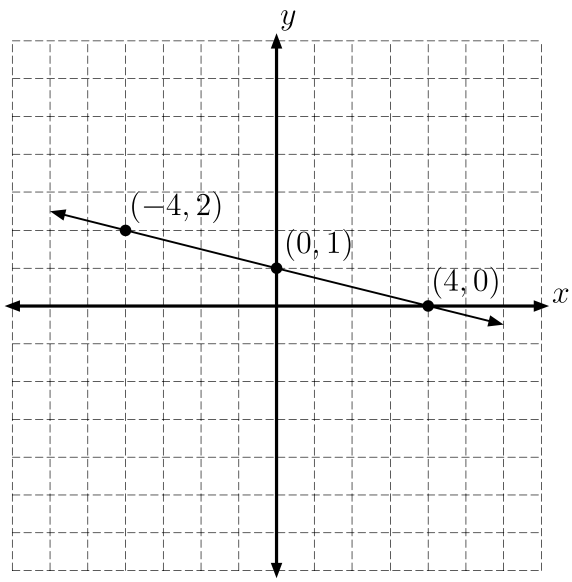


NOTE that the three points fall in a straight line. **EVERY** point on the line is a solution of the equation and can be represented by an ordered pair. Two points are sufficient to draw a straight line, but we generally get a third point as a check.

EXAMPLE: Graph $y = -\frac{1}{4}x + 1$

NOTE that the coefficient of *x* in this equation is a fraction. When this occurs we want to pick values for *x* which will allow us to eliminate the fraction. As the denominator of the fraction is 4, the easiest choices to work with will be multiples of 4, such as 0, 4 and –4.

Let $x = 0$:	Let $x = 4$:	Let $x = -4$:
$y = -\frac{1}{4}(0) + 1$	$y = -\frac{1}{4}(4) + 1$	$y = -\frac{1}{4}(-4) + 1$
$y = 0 + 1$	$y = -1 + 1$	$y = 1 + 1$
$y = 1$	$y = 0$	$y = 2$
(0, 1)	(4, 0)	(–4, 2)



Any time the coefficient of *x* is a fraction, convenient choices for *x* are zero, the denominator, and the opposite of the denominator.

$y = -\frac{2}{3}x + 8$ choose 0, 3, and –3

$y = -\frac{2}{5}x - 4$ choose 0, 5, and –5

Sometimes the equation is in the form of Ax + By = C, and in this case we can solve the equation for *y* first.

EXAMPLE: Graph $2x+3y=12$

To solve the equation for *y*, follow these steps:

1. To isolate the *y* term, add the opposite of the term containing *x* to both sides of the equation.

$$2x+3y=12$$
$$2x-2x+3y=-2x+12$$
$$3y=-2x+12$$

2. Divide both sides of the equation by the coefficient of y. This means **<u>both terms</u>** on the right-hand side must be divided by the coefficient.

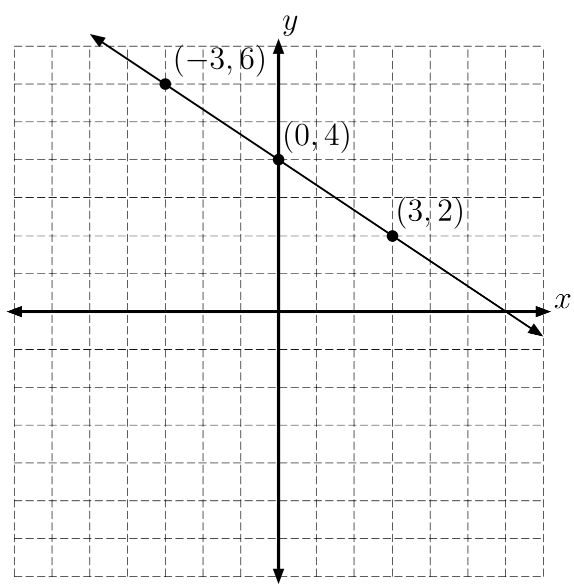
$$\frac{3y}{3}=\frac{-2x}{3}+\frac{12}{3}$$
$$y=-\frac{2}{3}x+4$$

Once the equation is in the form of $y = mx + b$, the ordered pair solutions can be found by picking values for *x* and solving for *y*. As the coefficient of *x* is $-\frac{2}{3}$ we would pick 0, 3 and –3 to get the ordered pairs.

Let $x = 0$:
$$y=-\frac{2}{3}(0)+4$$
$$y=0+4$$
$$y=4$$
(0, 4)

Let $x = 3$:
$$y=-\frac{2}{3}(3)+4$$
$$y=-2+4$$
$$y=2$$
(3, 2)

Let $x = -3$:
$$y=-\frac{2}{3}(-3)+4$$
$$y=2+4$$
$$y=6$$
(–3, 6)



Now we will practice rewriting equations in the form Ax + By = C to their equivalent $y = mx + b$ form.

EXAMPLE: Solve for y: $x + 4y = -6$

$$x + (-x) + 4y = -x - 6 \qquad \text{Add } -x \text{ to both sides}$$
$$4y = -x - 6$$
$$\frac{4}{4}y = -\frac{x}{4} - \frac{6}{4} \qquad \text{Divide each term by 4}$$
$$y = -\frac{1}{4}x - \frac{3}{2}$$

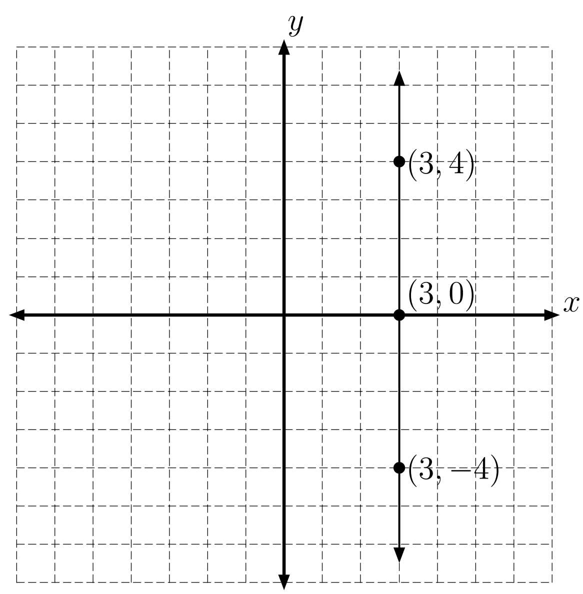
EXAMPLE: Solve for y: $-2x - 4y = 8$

$$-2x + 2x - 4y = 2x + 8 \qquad \text{Add } 2x \text{ to both sides}$$
$$-4y = 2x + 8$$
$$\frac{-4}{-4}y = \frac{2x}{-4} + \frac{8}{-4} \qquad \text{Divide each term by } -4$$
$$y = -\frac{1}{2}x - 2$$

EXAMPLE: Graph the equation $x = 3$.

NOTICE that the equation $x = 3$ does not mention y. This equation could be written as $0 \cdot y + x = 3$. In this case no matter what value y has, because y is multiplied by 0, x will always be 3. This graph will be a **vertical line** through the point where $x = 3$.

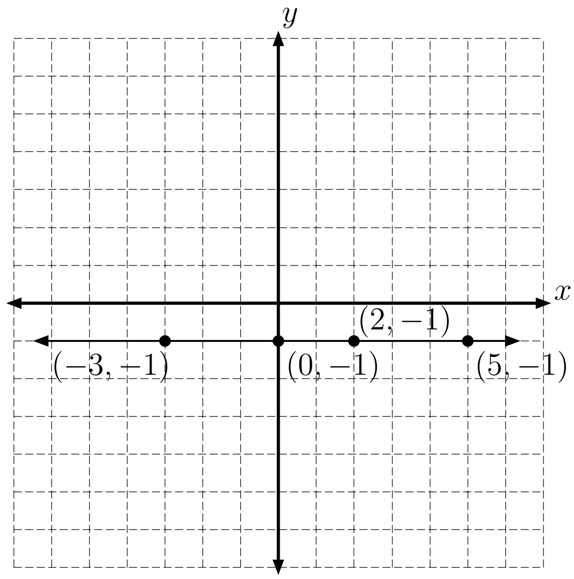
x	y
3	–4
3	–2
3	0
3	2
3	4



EXAMPLE: Graph the equation $y = -1$.

NOTICE that this equation does not mention x. This equation could be written as $0 \cdot x + y = -1$. In this case no matter what value x has, because x is multiplied by 0, y will always be –1. This graph will be a **horizontal line** through the point where $y = -1$.

x	y
–3	–1
–1	–1
0	–1
2	–1
5	–1



PRACTICE

Graph the lines of the following equations. Find at least three ordered pairs associated with each line.

a. $3x - y = 4$

b. $2x + y = 1$

c. $y = \frac{1}{2}x + 3$

d. $y = -\frac{1}{2}x + 3$

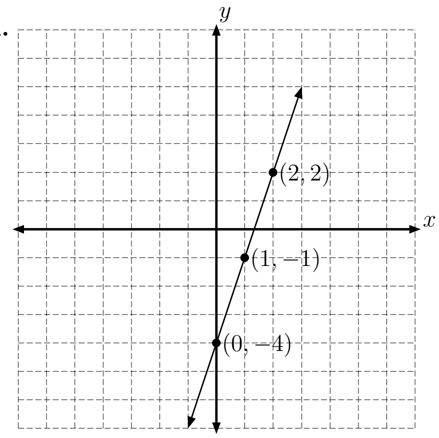
e. $2x + y = 4$

f. $3x + 4y = 12$

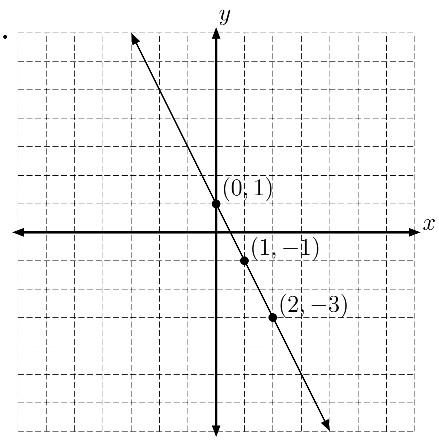
g. $x = -4$

h. $y = 3$

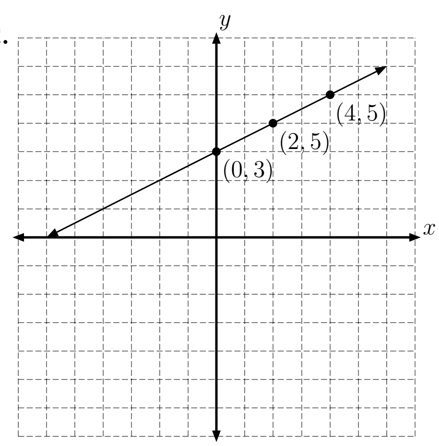
a.


y
x
(2, 2)
(1, −1)
(0, −4)

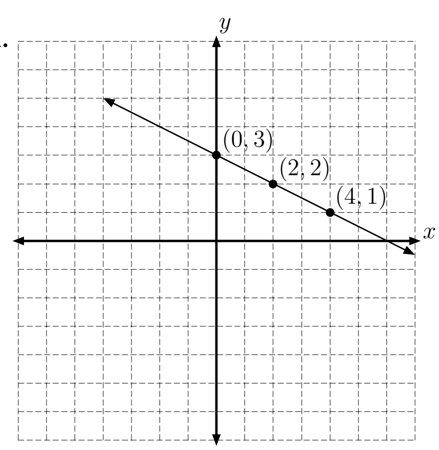
b.


y
x
(0, 1)
(1, −1)
(2, −3)

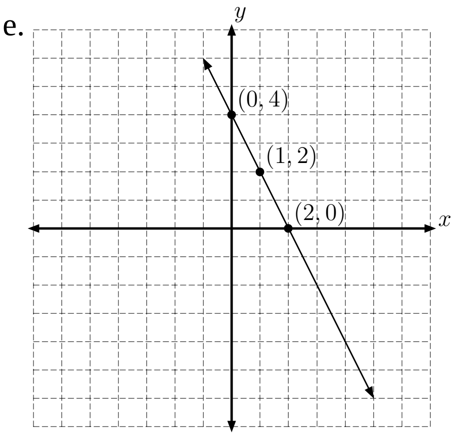
c.


y
x
(4, 5)
(2, 5)
(0, 3)

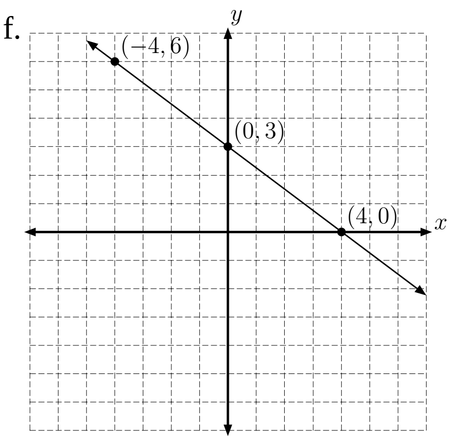
d.


y
x
(0, 3)
(2, 2)
(4, 1)

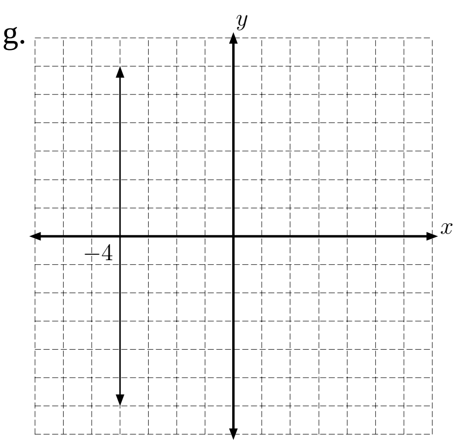
e.


y
x
(0, 4)
(1, 2)
(2, 0)

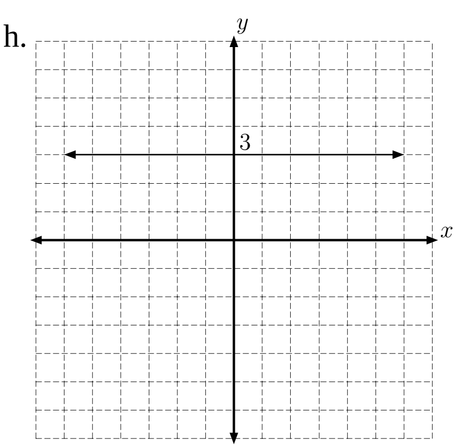
f.


y
x
(−4, 6)
(0, 3)
(4, 0)

g.


y
x
−4

h.


y
x
3