Graphing Linear Equations

A linear equation has infinitely many ordered pair solutions. The graph of an equation in two variables is a drawing of the ordered pair solutions of the equation. It is not possible to name all the solutions. We generally find three ordered pair solutions and graph them. The complete solution set can be shown by drawing a straight line through the graphs of the ordered pairs. An arrow on each end of the line shows that the solution set continues in both directions.

EXAMPLE: Graph \( y = 3x + 2 \)

To find three ordered pair solutions, pick any three values for \( x \) and solve for \( y \).

Let \( x = 1 \):
\[
\begin{align*}
y &= 3(1) + 2 \\
y &= 3 + 2 \\
y &= 5 \\
(1, 5)
\end{align*}
\]

Let \( x = -1 \):
\[
\begin{align*}
y &= 3(-1) + 2 \\
y &= -3 + 2 \\
y &= -1 \\
(-1, -1)
\end{align*}
\]

Let \( x = -2 \):
\[
\begin{align*}
y &= 3(-2) + 2 \\
y &= -6 + 2 \\
y &= -4 \\
(-2, -4)
\end{align*}
\]

Now we graph the ordered pair solutions \((1, 5), (-1, -1), \) and \((-2, -4)\).

NOTE that the three points fall in a straight line. EVERY point on the line is a solution of the equation and can be represented by an ordered pair. Two points are sufficient to draw a straight line, but we generally get a third point as a check.
EXAMPLE: Graph $y = -\frac{1}{4}x + 1$

**NOTE** that the coefficient of $x$ in this equation is a fraction. When this occurs we want to pick values for $x$ which will allow us to eliminate the fraction. As the denominator of the fraction is 4, the easiest choices to work with will be multiples of 4, such as 0, 4 and $-4$.

Let $x = 0$:  
\[
y = -\frac{1}{4}(0) + 1 \quad \Rightarrow \quad y = 1 \quad (0, 1)
\]

Let $x = 4$:  
\[
y = -\frac{1}{4}(4) + 1 \quad \Rightarrow \quad y = 0 \quad (4, 0)
\]

Let $x = -4$:  
\[
y = -\frac{1}{4}(-4) + 1 \quad \Rightarrow \quad y = 2 \quad (-4, 2)
\]

Any time the coefficient of $x$ is a fraction, convenient choices for $x$ are zero, the denominator, and the opposite of the denominator.

\[
y = -\frac{2}{3}x + 8 \quad \text{choose 0, 3, and } -3
\]

\[
y = -\frac{2}{5}x - 4 \quad \text{choose 0, 5, and } -5
\]

Sometimes the equation is in the form of $Ax + By = C$, and in this case we can solve the equation for $y$ first.
EXAMPLE: Graph $2x + 3y = 12$

To solve the equation for $y$, follow these steps:

1. To isolate the $y$ term, add the opposite of the term containing $x$ to both sides of the equation.

\[
2x + 3y = 12 \\
2x - 2x + 3y = -2x + 12 \\
3y = -2x + 12
\]

2. Divide both sides of the equation by the coefficient of $y$. This means **both terms** on the right-hand side must be divided by the coefficient.

\[
\frac{3y}{3} = \frac{-2x}{3} + \frac{12}{3} \\
y = -\frac{2}{3}x + 4
\]

Once the equation is in the form of $y = mx + b$, the ordered pair solutions can be found by picking values for $x$ and solving for $y$. As the coefficient of $x$ is $-\frac{2}{3}$ we would pick 0, 3 and $-3$ to get the ordered pairs.

Let $x = 0$: 

\[
y = -\frac{2}{3}(0) + 4 = 4
\]

Let $x = 3$:

\[
y = -\frac{2}{3}(3) + 4 = 2
\]

Let $x = -3$:

\[
y = -\frac{2}{3}(-3) + 4 = 6
\]

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Now we will practice rewriting equations in the form \(Ax + By = C\) to their equivalent \(y = mx + b\) form.

**EXAMPLE:** Solve for \(y\): \(x + 4y = -6\)

\[
x + (-x) + 4y = -x - 6
\]
\[
4y = -x - 6
\]
\[
\frac{4}{4}y = \frac{-x - 6}{4}
\]
\[
y = -\frac{1}{4}x - \frac{3}{2}
\]

**EXAMPLE:** Solve for \(y\): \(-2x - 4y = 8\)

\[
-2x + 2x - 4y = 2x + 8
\]
\[
-4y = 2x + 8
\]
\[
\frac{-4}{-4}y = \frac{2x + 8}{-4}
\]
\[
y = \frac{1}{2}x - 2
\]

**EXAMPLE:** Graph the equation \(x = 3\).

**NOTICE** that the equation \(x = 3\) does not mention \(y\). This equation could be written as \(0 \cdot y + x = 3\). In this case no matter what value \(y\) has, because \(y\) is multiplied by 0, \(x\) will always be 3. This graph will be a **vertical line** through the point where \(x = 3\).
EXAMPLE: Graph the equation \( y = -1 \).

**NOTICE** that this equation does not mention \( x \). This equation could be written as \( 0 \cdot x + y = -1 \). In this case no matter what value \( x \) has, because \( x \) is multiplied by 0, \( y \) will always be \(-1\). This graph will be a **horizontal line** through the point where \( y = -1 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-1</td>
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<tr>
<td>-1</td>
<td>-1</td>
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<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
</tr>
</tbody>
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**PRACTICE**
Graph the lines of the following equations. Find at least three ordered pairs associated with each line.

a. \( 3x - y = 4 \)  
e. \( 2x + y = 4 \)

b. \( 2x + y = 1 \)  
f. \( 3x + 4y = 12 \)

c. \( y = \frac{1}{2}x + 3 \)  
g. \( x = -4 \)

d. \( y = -\frac{1}{2}x + 3 \)  
h. \( y = 3 \)

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