Multiplying Polynomials

Multiplying a Polynomial by a Monomial

To multiply a polynomial by a monomial we use a Distributive Property as well as the rule for multiplying exponential expressions.

EXAMPLE: \(4x^2(x + 8)\)

We will first multiply \(4x^2\) and \(x\). Then we will multiply \(4x^2\) and 8.

\[
4x^2(x) + 4x^2(8) = 4x^3 + 32x^2
\]

REMEMBER to add the exponents if the bases are the same.

\[
4x^2(x) = 4x^{2+1} = 4x^3
\]

Also, REMEMBER that the sign we get when we multiply gives us the sign between the terms.

EXAMPLE: \(-y(-3y^2 - 2y + 6)\)

Use the Distributive Property to multiply each term inside the parentheses by \(-y\). REMEMBER that the sign in front of the term goes with the term.

\[
-y(-3y^2) - y(-2y) - y(6) = 3y^3 + 2y^2 - 6y
\]

REMEMBER that we cannot combine terms unless the variable parts are identical. This problem is simplified as far as possible.

EXAMPLE: \(ab(2a^2 - 4ab - 6b^2)\)

\[
ab(2a^3) + ab(-4ab) + ab(-6b^2) = 2a^3b - 4a^2b^2 - 6ab^3
\]

Don’t forget the rules for exponents!

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Multiplying a Polynomial by a Polynomial

Multiplication of polynomials can be accomplished by using a horizontal format and the Distributive Property, or by using a vertical format. We will use the vertical format—the process is similar to multiplication of real numbers.

EXAMPLE: \((y^2 - 2y + 7)(y - 2)\)

REWRITE in vertical format.

\[
\begin{array}{c}
y^2 - 2y + 7 \\
y - 2
\end{array}
\]

Multiply each term by \(-2\):

\[
\begin{array}{c}
y^2 - 2y + 7 \\
y - 2
\end{array}
= -2y^2 + 4y - 14
\]

Now multiply each term by \(y\). Be sure to keep like terms lined up.

\[
\begin{array}{c}
y^2 - 2y + 7 \\
y - 2
\end{array}
= y^3 - 2y^2 + 7y
\]

\[
\begin{array}{c}
y^3 - 2y^2 + 7y
\end{array}
= y^3 - 4y^2 + 11y - 14
\]

Now combine like terms.

If you compare multiplication of polynomials to long multiplication of integers you will see that the steps are very similar.

\[
\begin{array}{c}
379 \\
\times 23
\end{array}
= 3x^2 + 3x - 5
\]

Multiply by 3

\[
\begin{array}{c}
4x - 2
\end{array}
= -6x^2 - 6x + 10
\]

Multiply by \(-2\)

\[
\begin{array}{c}
379 \\
\times 23
\end{array}
= 3x^2 + 3x - 5
\]

Multiply by 2, keeping place values aligned

\[
\begin{array}{c}
4x - 2
\end{array}
= -6x^2 - 6x + 10
\]

Multiply by 4x, keeping like terms together

\[
\begin{array}{c}
12x^3 + 12x^2 - 20x
\end{array}
\]

\[
\begin{array}{c}
379 \\
\times 23
\end{array}
= 3x^2 + 3x - 5
\]

Now add to get the total.

\[
\begin{array}{c}
4x - 2
\end{array}
= -6x^2 - 6x + 10
\]

Add by combining like terms

\[
\begin{array}{c}
12x^3 + 12x^2 - 20x
\end{array}
= 12x^3 + 6x^2 - 26x + 10
\]
Sometimes there is a missing term in one of the polynomials. This means we must either leave a space or put in a place holder. We will use a place holder.

EXAMPLE: \((3a^3 - 5a^2 + 7)(6a - 1)\)

Notice that the first polynomial has no “a” term. We’ll hold that place with “0a.”

\[
\begin{array}{c}
3a^3 - 5a^2 + 0a + 7 \\
\hline
-3a^3 + 5a^2 - 0a - 7 \\
\hline
18a^4 - 30a^3 + 0a^2 + 42a \\
\hline
18a^4 - 33a^3 + 5a^2 + 42a - 7
\end{array}
\]

\(\leftarrow\) multiply by \(-1\)

\(\leftarrow\) multiply by \(6a\)

\(\leftarrow\) add

EXAMPLE: \((5x^3 - 5x + 2)(x - 4)\)

There is no “x^2” term in the first polynomial.

\[
\begin{array}{c}
5x^3 + 0x^2 - 5x + 2 \\
\hline
-20x^3 + 0x^2 + 20x - 8 \\
\hline
5x^4 + 0x^3 - 5x^2 + 2x \\
\hline
5x^4 - 20x^3 - 5x^2 + 22x - 8
\end{array}
\]

\(\leftarrow\) multiply by \(-4\)

\(\leftarrow\) multiply by \(x\)

\(\leftarrow\) add

EXERCISES: Multiply each of the following.

a. \(-6a^3(a + 2)\)  
f. \((-x^2 + 3x - 2)(2x - 1)\)

b. \(2y(3y^2 - 4)\)  
g. \((3y^2 + 2y - 2)(5y - 4)\)
c. \(-x(-2x^4 - 3x^2 + 2)\)  

h. \((-2x^2 + 3x + 8)(x + 7)\)

d. \((3y^3 - 2y^2 + 5)4y\)  
i. \((4a^3 - 2a + 5)(a + 6)\)

e. \(-3ab(a^3 - 5ab + 3b^2)\)  
j. \((5y^3 + 2y^2 - 9)(2y - 3)\)

KEY:
a. \(-6a^4 - 12a^3\)  
b. \(6y^3 - 8y\)  
c. \(2x^5 + 3x^3 - 2x\)  
d. \(12y^4 - 8y^3 + 20y\)  
e. \(-3a^3b + 15a^2b^2 - 9ab^3\)  
f. \(-2x^3 + 7x^2 - 7x + 2\)  
g. \(15y^3 - 2y^2 - 18y + 8\)  
h. \(-2x^3 - 11x^2 + 29x + 56\)  
i. \(4a^4 + 24a^3 - 2a^2 - 7a + 30\)  
j. \(10y^4 - 11y^3 - 6y^2 - 18y + 27\)