Systems of Linear Equations

EXAMPLE

1. Solve for the system of linear equations.

\[\begin{align*}
4x + 2y &= -6 \\
5x + 5y &= 10
\end{align*}\]

SOLUTION

We will eliminate \( y \).

Multiply the first equation by \(-5\) and multiply the second equation by \(2\).

\[\begin{align*}
-20x - 10y &= 30 \\
10x + 10y &= 20
\end{align*}\]

Add the two equations.

\[\begin{align*}
-10x &= 50
\end{align*}\]

Solve for \( x \).

\[x = -5\]

Let \( x = -5 \) in \(4x + 2y = -6\).

\[4(-5) + 2y = -6\]

\[-20 + 2y = -6\]

Solve for \( y \).

\[2y = 14\]

\[y = 7\]

The solution occurs when \( x = -5, y = 7 \).

The solution is \((-5, 7)\)

The system is consistent and independent system.
EXAMPLE

2. Solve for the system of linear equations.

\[ 12x - 4y = 20 \]
\[ 9x - 3y = 5 \]

SOLUTION

We will eliminate \( y \).

Multiply the first equation by \(-3\) and multiply the second equation by \(4\).

\[ -36x + 12y = -60 \]
\[ 36x - 12y = 20 \]

Add the two equations.

\[ 0 = -40 \]

Since use of the addition method has resulted in the case where all of the variables cancel and we are left with a false statement \((0 = -40)\), we know that the system of equations has no solution.

This is an inconsistent system.
EXAMPLE

3. Solve for the system of linear equations

\[
\begin{align*}
x - \ 5y &= \ 3 \\
-4x + 20y &= -12
\end{align*}
\]

SOLUTION

\[
\begin{align*}
x - \ 5y &= \ 3 \\
-4x + 20y &= -12
\end{align*}
\]

We will eliminate \(x\).

Multiply the first equation by 4.

\[
\begin{align*}
4x - 20y &= 12 \\
-4x + 20y &= -12
\end{align*}
\]

Add the two equations.

\[
0 = 0
\]

Since use of the addition method has resulted in the case where all of the terms cancel on both sides of the equals sign, leaving the true statement \(0 = 0\), we know that there are infinitely many solutions.

\[
\{(x, y) | x - 5y = 3\}
\]

This is a \textbf{consistent} and \textbf{dependent} system.
Exercises: Solve for the following systems of linear equations

1. \[\begin{align*}
3x + 2y &= -1 \\
2x - 3y &= -5
\end{align*}\]

2. \[\begin{align*}
5x - 2y &= 5 \\
-10x + 4y &= -8
\end{align*}\]

3. \[\begin{align*}
2x - 4y &= -10 \\
5x + 4y &= 17
\end{align*}\]

4. \[\begin{align*}
5x - 2y &= 5 \\
-10x + 4y &= -10
\end{align*}\]

5. \[\begin{align*}
3x - 4y &= 5 \\
-3x + 4y &= -3
\end{align*}\]

Answers:
1. \((-1,1)\)  \quad 2. \(\emptyset\)  \quad 3. \((1,3)\)  \quad 4. \(\{(x, y) | 5x - 2y = 5\}\)  \quad 5. \(\emptyset\)