Independent, Dependent and Mutually Exclusive Events

INDEPENDENCE (”not affecting”)

Two events are independent if the occurrence of one does not affect the probability of the occurrence of the other.

Example: Consider the events of flipping a coin and tossing a six-sided die.

The probability of getting heads in a single toss of a fair coin is 1/2. The probability of getting a four in a single roll of a fair die is 1/6 regardless of the results of the coin toss. The two experiments are independent because the result of the coin toss does not affect the probability of the outcome of the die roll.

DEPENDENCE (”affecting”)

Two single events are dependent if the occurrence of one does affect the probability of the occurrence of the other.

Example: Four different colored balls are in a bag. Two balls are to be drawn from the bag in succession (without replacing the first ball before the second ball is drawn). Consider the events of pulling a blue ball out on the first draw and pulling a blue ball out on the second draw.

The probability that the first ball drawn is blue is 1/4. Suppose the red ball is drawn first. The probability the second ball will be blue is 1/3. On the other hand, if the first ball drawn is blue, the probability the second ball will be blue is 0 because there would be no blue balls left in the bag. The two events, pulling a blue ball out on the first draw and pulling a blue ball out on the second draw, are dependent since the outcome of the first draw does affect the probability of drawing a blue ball on the second draw.
Mutually Exclusive Events (“not at the same time”)

For a single experiment, two events are said to be mutually exclusive if they cannot occur at the same time.

Example: A single card is drawn from a standard 52-card deck. Drawing a card that is simultaneously a red card and a club cannot occur because these events are mutually exclusive.

![Venn diagram showing mutually exclusive events]

Example: a 5 of clubs is drawn, it is not a red card. As shown above in the Venn diagram, the sets of red cards and clubs are distinct and separate; therefore the two events are mutually exclusive.

Not Mutually Exclusive Events (“at the same time”)

Example: A single card is drawn from a standard 52-card deck. Drawing a card that is both black and a club is possible. The two events are not mutually exclusive because they can occur simultaneously.

![Venn diagram showing not mutually exclusive events]

If a club is drawn, it is also a black card since all clubs are black. Therefore the two events are not mutually exclusive.

Example: A single card is drawn from a standard 52-card deck. Drawing a card that is both a five and a black card is possible. The two events are not mutually exclusive because they can occur simultaneously.

![Venn diagram showing not mutually exclusive events]

The five of clubs and the five of spades are simultaneously fives and black cards. Thus, the events drawing a club and drawing a five are not mutually exclusive.