Vector Algebra

Vector notation is designed to not just give you a point in space but a directional vector. They will usually look like $<2,5>$. This is giving you a vector that starts at the origin and continues in the direction of the $(x, y)$ point of $(2,5)$. Vectors can also be written in the form: $2i + 5j$. The $i$ component is always $x$ and the $j$ is $y$.

**Basic Vector Operations and Rules** (let $u = <a,b>$ and $v = <c,d>$)

- Magnitude of a vector: $|v| = \sqrt{a^2 + b^2}$; length of the vector
- Addition Rules: $u + v = <a,b> + <c,d> = <a + c, b + d>$
- Scalar Multiplication: $cu = c<a,b> = <ca, cb>$

Examples of addition and subtraction with vectors, scalar multiplication:

**Let:** $U = <2,3>$, $W = <-4, 6>$

1) $U + W = <2 - 4, 3 + 6> = <-2, 9>$ or $-2i + 9j$
2) $W - U = <-4, 6> - <2,3> = <-4 - 2, 6 - 3> = <-6, 3>$ or $-6i + 3j$
3) $2U - 3W = <4, 6> - <-12, 18> = <4 - (-12), 6 - 18> = <16, -12>$ or $16i - 12j$

You Try (1): find $4U - 2W$

**Unit Vector- magnitude is 1**

Example of converting a vector to a unit vector: $v = <2,5>$

1) Find the magnitude $\rightarrow \sqrt{2^2 + 5^2} = \sqrt{4 + 25} = \sqrt{29}$
2) $U = \frac{1}{|v|}v = <\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}>$
3) This would give a unit vector (length of 1) in the direction of $v$.

You Try(2): find the unit vector for the vector $v = <-1, 5>$

**Dot Product**

The dot product is used to find the angle between two vectors. The result will always be a scalar (a number not a vector).

Definition: if $u = <a,b>$ and $v = <c,d>$ then $u \cdot v = ac + bd$

Example: $u = <2,3>$, $v = <-4, 6>$, then $u \cdot v = 2 \times -4 + 3 \times 6 = -8 + 18 = 10$

You Try(3): $u = <1, 5>$, $v = <-2, 1>$ solve for $u \cdot v$
Find the angle between two vectors (application of the dot product)

\[ \cos \theta = \frac{u \cdot v}{|u||v|} \]

So if \( U = <2,3>, \ V = <-4, 6> \), then:

\[ \cos \theta = \frac{<2,3> \cdot <-4,6>}{\sqrt{13}\sqrt{52}} = \frac{-8 + 18}{26} = \frac{10}{26} = \frac{5}{13}; \text{ thus } \cos \theta = \frac{5}{13}; \text{ so } \theta = 67.38^\circ. \]

On another note, two vectors are said to “orthogonal” (at a right angle or 90° apart) if \( u \cdot v = 0 \)

You Try(4): find the angle between \( U = <-2,1> \) and \( V = <-4, 3> \)

**Finding the scalar component of \( u \) on \( v \)**

An important use of the dot product is to determine the projection of one vector onto another if they share a common initial point.

If \( \theta \) is the angle between vectors \( u \) and \( v \), then the scalar component of \( u \) on \( v \) is given by:

\[ \text{comp}_v u = \frac{u \cdot v}{|v|} \]

So if \( u = <2,3>, \ v = <-4, 6> \), then

\[ \text{comp}_v u = \frac{<2,3> \cdot <-4,6>}{\sqrt{13}} = \frac{10}{\sqrt{13}} = 2.77 \text{ to 2 significant figures.} \]

One application of this is Work which can be defined as:

\[ \text{comp}_d F \ |d| = \frac{F \cdot d}{|d|} \ |d| = F \cdot d \]

**Answers to You Try’s:** 1) \( <16, 0> \) 2) \( U = \left< -\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}} \right> \) 3) \( 3 \) 4) \( 26.57^\circ \)

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