

## Cost, Revenue and Profit Functions

Earl's Biking Company manufactures and sells bikes. Each bike costs \$40 to make, and the company's fixed costs are \$5000. In addition, Earl knows that the price of each bike comes from the price function  $p(x) = 300 - 2x$ . Find:

1. The company's revenue function,  $R(x)$ .
2. The company's cost function,  $C(x)$ .
3. The company's profit function,  $P(x)$ .
4. The two points,  $(x_1, y_1)$  and  $(x_2, y_2)$  at which the company breaks even.
5. The output level that maximizes the company's profit, and the maximum profit.

**1)** Revenue is equal to the number of units sold times the price per unit. To obtain the revenue function, multiply the output level by the price function.

$$\begin{aligned}R(x) &= x \cdot p(x) \\ &= x \cdot (300 - 2x) \\ &= \mathbf{300x - 2x^2}\end{aligned}$$

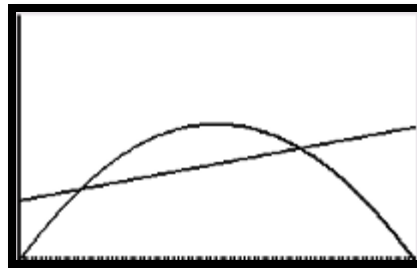
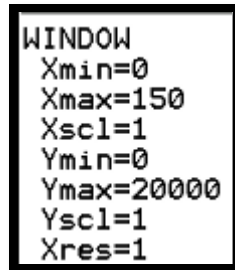
**2)** A business' costs include the fixed cost of \$5000 as well as the variable cost of \$40 per bike. To obtain the cost function, add fixed cost and variable cost together.

$$\begin{aligned}C(x) &= (\text{fixed cost}) + x \cdot (\text{variable cost}) \\ &= (5000) + x \cdot (40) \\ &= \mathbf{5000 + 40x}\end{aligned}$$

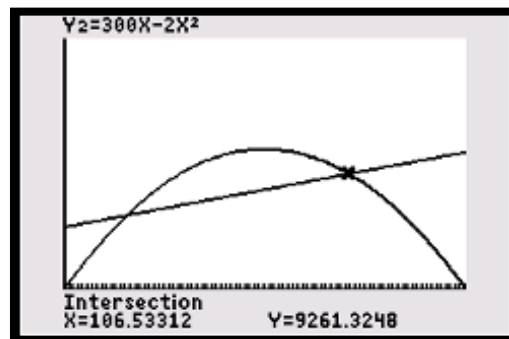
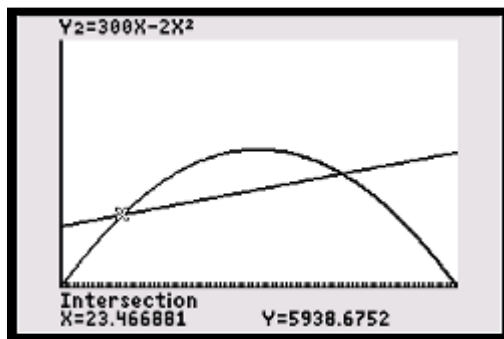
**3)** The profit a business makes is equal to the revenue it takes in minus what it spends as costs. To obtain the profit function, subtract costs from revenue.

$$\begin{aligned}P(x) &= R(x) - C(x) \\ &= (300x - 2x^2) - (5000 + 40x) \\ &= 300x - 2x^2 - 5000 - 40x \\ &= \mathbf{-2x^2 + 260x - 5000}\end{aligned}$$

**4)** A company's break-even points occur where the revenue function and the cost function have the same value. This also implies that the profit function equals zero at break-even points. These points are found most easily on a graphing calculator. Use the graphing tools to plot cost and revenue functions, and find a suitable window (it may be helpful to use ZOOM, ZOOMFIT).



The graph shows two points where the linear cost function and the parabolic revenue function intersect. Use 2ND, CALC, INTERSECT to find their coordinates, and round to the nearest integer as needed.



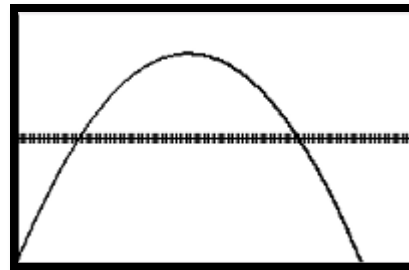
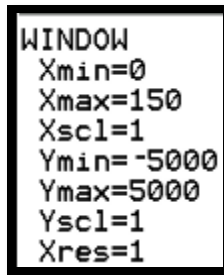
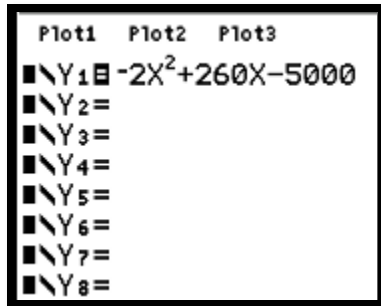
5) If you choose to find the output level that maximizes profit by hand, use the formula  $x = \frac{-b}{2a}$  to find the vertex of the profit function,  $P(x)$ .

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-(260)}{2(-2)} \\ &= 65 \end{aligned}$$

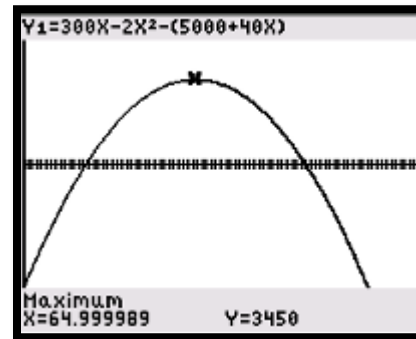
Then, to find the profit generated from this output level, substitute this x-value into  $P(x)$ .

$$\begin{aligned} y &= P(65) \\ &= \mathbf{3450} \end{aligned}$$

To find these values in the calculator, plot the profit function  $P(x)$  in the same way as was outlined in part 4).



Using 2<sup>ND</sup>, CALC, MAXIMUM, the output level can be found to be the x-coordinate of the vertex ( $x = 65$ ), and the profit generated can be found to be the y-coordinate of the vertex ( $y = 3450$ ).



**Examples:** Repeat steps 1-5 as demonstrated for the following scenarios:

- A manufacturer of handheld games, with fixed costs of \$15000, cost per game of \$2, and price function  $p(x) = 500 - 3x$ .
- A chocolatier, with fixed costs of \$300, cost per box of chocolates \$4, and price function  $p(x) = 18 - 0.1x$ .

Answers:

**A**

- $R(x) = 500x - 3x^2$
- $C(x) = 15000 + 2x$
- $P(x) = -3x^2 + 498x - 15000$
- (40, 15079) and (126, 15253)
- Output = 83 games, Profit = \$5667.

**B**

- $R(x) = 18x - 0.1x^2$
- $C(x) = 300 + 4x$
- $P(x) = -0.1x^2 + 14x - 300$
- (26, 406) and (114, 754)
- Output = 70 boxes, Profit = \$190.