## Tests for Convergence of Series

### Test for Divergence

If \( \lim_{n \to \infty} a_n \neq 0 \) or fails to exist, then \( \sum_{n=1}^{\infty} a_n \) diverges.

### Geometric Series

For \( \sum_{n=1}^{\infty} ar^{n-1} \), the series converges for \(|r| < 1\), \( s = \frac{a}{1-r} \).

- Diverges for \(|r| \geq 1\).

### p Series

For \( \sum_{n=1}^{\infty} \frac{1}{n^p} \), the series converges for \( p > 1 \).

- Diverges for \( p \leq 1 \).

### Ratio Test

If \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L \),

- i. The series is Absolutely convergent if \( L < 1 \) and therefore is convergent.
- ii. The series diverges if \( L > 1 \) or is infinite.
- iii. The test is inconclusive if \( L = 1 \).

### Root Test

If \( \lim_{n \to \infty} \sqrt[n]{|a_n|} = L \),

- The series is Absolutely convergent if \( L < 1 \) and therefore is convergent.
- The series diverges if \( L > 1 \) or is infinite.
- The test is inconclusive if \( L = 1 \).

### Absolute Convergence

If \( \sum_{n=1}^{\infty} |a_n| \) converges, then \( \sum_{n=1}^{\infty} a_n \) converges absolutely and is convergent.

### Integral Test

Suppose \( f \) is a continuous, positive, decreasing function on \([1, \infty)\) and let \( a_n = f(n) \).

- i) If \( \int_{1}^{\infty} f(x) \, dx \) is convergent, then \( \sum_{n=1}^{\infty} a_n \) is convergent.
- ii) If \( \int_{1}^{\infty} f(x) \, dx \) is divergent, then \( \sum_{n=1}^{\infty} a_n \) is divergent.

### The Limit Comparison Test

Suppose \( \sum a_n \) and \( \sum b_n \) are series with positive terms.

- If \( \lim_{n \to \infty} \frac{a_n}{b_n} = c \), where \( c \) is a finite number and \( c > 0 \) then both series converge or both series diverge.

### The Comparison Test

Suppose \( \sum a_n \) and \( \sum b_n \) are series with positive terms.

- i) If \( \sum b_n \) is convergent and \( a_n \leq b_n \) for all \( n \), then \( \sum a_n \) is also convergent.
- ii) If \( \sum b_n \) is divergent and \( a_n \geq b_n \) for all \( n \), then \( \sum a_n \) is also divergent.

### Alternating Series Test

If the alternating series \( \sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \cdots \) \( b_n > 0 \) satisfies

- i) \( b_{n+1} \leq b_n \quad \forall n \)
- ii) \( \lim_{n \to \infty} b_n = 0 \) then the series is convergent.