Verifying trigonometric identities

Process: make one side look exactly like the other using a combination of trigonometric identities and algebra. You can work with only one side at a time.

1. **Algebra techniques utilized**
   a. “FOIL”ing example 1
      \[(\cot x - \csc x)(\cos x + 1) = -\sin x\]
   b. “FOIL”ing example 2
      \[\frac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t\]
   c. distribution
      \[\sec t \csc t (\tan t + \cot t) = \sec^2 t + \csc^2 t\]
   d. Common denominator
      \[2 \sec x = \frac{1}{\sec x + \tan x} + \frac{1}{\sec x - \tan x}\]

2. **Conjugate**
   \[\frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}\]

3. **Substitution of identity**
   \[\sin^2 x + \cos^2 x + \tan^2 x = \sec^2 x\]

4. **Turn all functions into \(\sin x\) and \(\cos x\)**
   \[\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1\]

If all else fails, turn everything into sine \(x\) and cosine \(x\) and see what happens! Usually there is lots of algebra between using the trig functions. You have to be very familiar with the basic functions.

### Basic Functions

\[
\begin{align*}
\sec x &= \frac{1}{\cos x} & \csc x &= \frac{1}{\sin x} & \cot x &= \frac{1}{\tan x} & \tan x &= \frac{\sin x}{\cos x} \\
\sin^2 x + \cos^2 x &= 1 & 1 + \cot^2 x &= \csc^2 x & \tan^2 x + 1 &= \sec^2 x
\end{align*}
\]

The last two can be obtained by dividing the first either by sine squared \(x\) or cosine squared \(x\). Might also look like cosine \(x = 1\) minus sine squared \(x\) or \(1 = \secant squared x - \tan x\) squared \(x\).
Examples

Worked out (remember, work with only one side until it looks like the other)

1. \((\cot x - \csc x)(\cos x + 1) = -\sin x\)

   \[
   \begin{align*}
   &= \cot x \cos x + \cot x - \csc x \cos x - \csc x \\
   &= \frac{\cos x}{\sin x} \cos x + \frac{\cos x}{\sin x} - \frac{1}{\sin x} \cos x - \frac{1}{\sin x} \\
   &= \frac{\cos^2 x}{\sin x} + \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x} - \frac{1}{\sin x} \\
   &= \frac{\cos^2 x - 1}{\sin x} \\
   &= \frac{(1 - \sin^2 x) - 1}{\sin x} \\
   &= -\frac{\sin^2 x}{\sin x} = -\sin x \quad \checkmark
   \end{align*}
   \]

   (working with left side since more complicated)

   FOIL the binomials

   insert \(\sin x / \cos x\) identities

   simplify

   cancel like terms

   identity; eliminate \(\cos x\) term since not in answer

   reduce

2. \(\frac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t\)

   \[
   \begin{align*}
   &= \frac{\sin^2 t + 2 \sin t \cos t + \cos^2 t}{\sin t \cos t} \\
   &= \frac{1 + 2 \sin t \cos t}{\sin t \cos t} \\
   &= \frac{1}{\sin t \cos t} + \frac{2 \sin t \cos t}{\sin t \cos t} \\
   &= \csc t \sec t + 2 \quad \checkmark
   \end{align*}
   \]

   (working with left side since more complicated)

   FOIL out the top

   combine \(\sin^2 x + \cos^2 x = 1\)

   separate fraction since final answer doesn’t have one

   use reciprocals and reduce fraction